NATIONAL MUSEUM OF THE UNITED STATES AIR FORCE

MATHEMATICS OF FLIGHT: HEIGHT OF AN AIRCRAFT

Students will have a basic understanding of math applications used in flight. This includes height of an aircraft in flight. Students will solve a series of problems. (One in a series)

LESSON PLAN

Lesson Objectives

The students will:

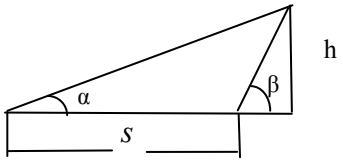
- Be introduced to formulas used in flight related to navigation and aircraft performance.
- Learn to calculate the height of an aircraft in flight.

Goal

In this lesson, students will gain an understanding of common calculations performed by flight personnel.

Calculating the Height of an Aircraft in Flight

One formula used to determine the height on an aircraft in flight being observed by two tracking stations, when given the distance between tracking stations (s) and the angles of inclination (α and β) is:



$$h = \frac{s}{\cot \alpha - \cot \beta}$$

See example on the following page.

Grade Level: 9-12

National Mathematics Content Standards:
Number and Operations: Understand meanings of operations and how they relate to one another. Algebra: Represent and analyze mathematical situations and structures using algebraic symbols; Use mathematical models to represent and understand quantitative relationships. Geometry: Use visualization, spatial reasoning , and geometric modeling to solve problems.

Technology Content Standards (from STL):
Technology and Society.

Materials Required:

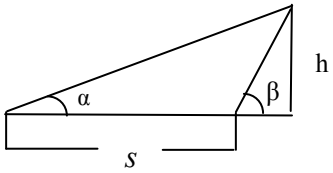
- Paper
- Pencil or Pen
- Trigonometry Tables
- Formula:

$$\mathbf{h = \frac{s}{\cot \alpha - \cot \beta}}$$

Lockheed F-94C Starfire at the National Museum of the United States Air Force. (U.S. Air Force photo)

Example:
Two tracking stations 12.95 miles apart observe an aircraft flying over them. How high, ***in feet,*** is the aircraft above the ground if the plane's angle of inclination at the first station is 30° and the angle at the second tracking station is 70° ?

Solution:



$$h = \frac{s}{\cot \alpha - \cot \beta}$$

$$h = \frac{12.95}{\cot 30 - \cot 70}$$

$$h = \frac{12.95}{1.7320 - 0.3640}$$

$$h = \frac{12.95}{1.3680}$$

$$h = 9.4664$$

The aircraft's height is 9.4664 miles. To convert this to feet, we know that one mile equals 5,280 feet.

$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{9.4664}{h}$$

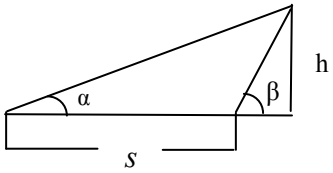
$h = 9.4664 \times 5{,}280$
$h = 49{,}982.46$ feet

The aircraft is 49,982 feet above the ground.

Lockheed F-94C Starfire
The F-94 series all-weather interceptors were developed from the Lockheed P-80 Shooting Star. The prototype F-94 first flew on July 1, 1949. Improvements in the F-94C included a higher thrust engine, single point refueling, a redesigned wing, a sweptback horizontal stabilizer, upgraded fire-control and navigation systems, and later, mid-wing rocket pods. Twenty-four rockets were carried in the nose in a ring around the radome, shielded by retractable doors, with an additional 24 in the wing pods, if installed. The F-94C carried no guns. Starfires were employed in the air defense of the continental United States in the 1950s. It had a ceiling of 51,800 feet.

Exercise 1:
Two tracking stations 20.6 miles apart observe an aircraft flying over them. How high, ***in feet,*** is the aircraft above the ground if the plane's angle of inclination at the first station is 20° and the angle at the second tracking station is 60° ?

Solution:



$$h = \frac{s}{\cot \alpha - \cot \beta}$$

$$h = \frac{20.6}{\cot 20 - \cot 60}$$

$$h = \frac{20.6}{2.7475 - 0.5774}$$

$$h = \frac{20.6}{2.1701}$$

$$h = 9.4926$$

The aircraft's height is 9.4926 miles. To convert this to feet, we know that one mile equals 5,280 feet.

$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{9.4926}{h}$$

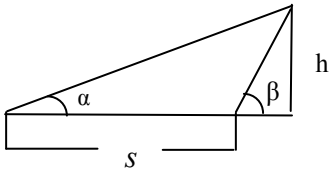
$h = 9.4926 \times 5{,}280$
$h = 50{,}120$ feet

The aircraft is 50,120 feet above the ground.

Convair F-106A Delta Dart
This all-weather interceptor had a ceiling of 53,000 ft. The first F-106A flew on December 26, 1956, and deliveries to the Air Force began in July 1959. The F-106 used a Hughes MA-1 electronic guidance and fire control system. After takeoff, the MA-1 can be given control of the aircraft to fly it to the proper altitude and attack position. Then it can fire the Genie and Falcon missiles, break off the attack run and return the aircraft to the vicinity of its base. The pilot takes control again for the landing.

Exercise 2:
An aircraft is flying at 10,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°?

Solution:



$$h = \frac{s}{\cot \alpha - \cot \beta}$$

$$10{,}000 = \frac{s}{\cot 10 - \cot 80}$$

$$10{,}000 = \frac{s}{5.6713 - 0.1763}$$

$$10{,}000 = \frac{s}{5.495}$$

$$5.4950 \text{ X } 10{,}000 \text{ ft} = \frac{s}{5.495} \text{ X } 5.4950$$

$$54{,}950 \text{ ft} = s$$

The distance is 54,950 ft.

$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{s}{54{,}950 \text{ feet}}$$

$$s = \frac{54{,}950}{5{,}280}$$

$$s = 10.4017 \text{ miles}$$

The tracking stations are 10.4017 miles apart.

Nieuport 28

The French-built Nieuport 28 became the first fighter airplane flown in combat by pilots of the American Expeditionary Force (AEF) in World War I. The lightly built Nieuport 28 developed a reputation for shedding its upper wing fabric in a dive, and by the spring of 1918, many considered the Nieuport 28 obsolete. Even so, American pilots maintained a favorable ratio of victories to losses with it. Many American aces of WWI, including 26-victory ace Capt. Eddie Rickenbacker, flew the Nieuport at one time or another in their careers. It had a ceiling of 17,000 ft.

See student worksheet and presentation
Examples are from the collection of the National Museum of the U.S. Air Force

Resources:

National Museum of the United States Air Force

- http://www.nationalmuseum.af.mil/factsheets/factsheet.asp?id=364
- http://www.nationalmuseum.af.mil/factsheets/factsheet.asp?id=4085
- http://www.nationalmuseum.af.mil/factsheets/factsheet.asp?id=273

Belcher, Diana. *Education in Flight: A Teacher's Guide to the Mathematics of Flight.* Department of the Air Force, 2007.

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STUDENT WORKSHEET *NAME:*_____

Exercise 1:
Two tracking stations 20.6 miles apart observe an aircraft flying over them. How high, ***in feet,*** is the aircraft above the ground if the plane's angle of inclination at the first station is 20° and the angle at the second tracking station is 60° ?

$$h = \frac{s}{\cot \alpha - \cot \beta}$$

Exercise 2:
An aircraft is flying at 10,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°? (One mile = 5,280 feet)

$$h = \frac{s}{\cot \alpha - \cot \beta}$$