Mathematics of Flight

Height of an Aircraft in Flight

Height of an Aircraft in Flight

To determine the height of an aircraft in flight being observed by two tracking stations, the following formula can be used:

$$h = \frac{s}{\cot \alpha - \cot \beta}$$

when given the distance between tracking stations (s) and the angles of inclination $(\alpha \text{ and } \beta)$

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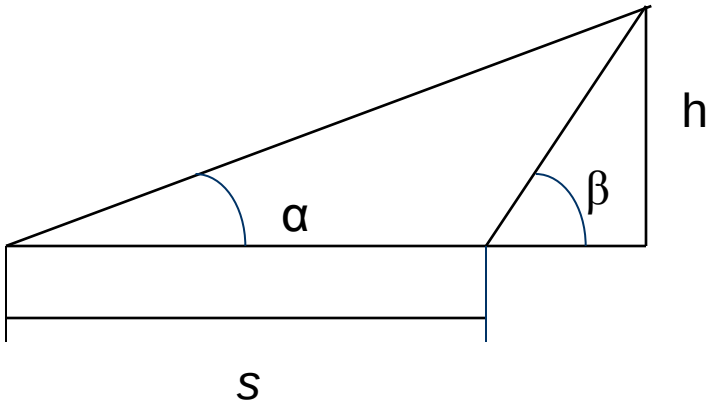

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The F-94 series all-weather interceptors were developed from the Lockheed P-80 Shooting Star. The prototype F-94 first flew on July 1, 1949. Improvements in the F-94C included a higher thrust engine, single point refueling, a redesigned wing, a sweptback horizontal stabilizer, upgraded fire-control and navigation systems, and later, mid-wing rocket pods. Twenty-four rockets were carried in the nose in a ring around the radome, shielded by retractable doors, with an additional 24 in the wing pods, if installed. The F-94C carried no guns. Starfires were employed in the air defense of the continental United States in the 1950s. The last F-94Cs were withdrawn from USAF service in 1959.

Ceiling: 51,800 ft.

Height of an Aircraft in Flight

Example:
Two tracking stations 12.95 miles apart observe an aircraft flying over them. How high, ***in feet,*** is the aircraft above the ground if the plane's angle of inclination at the first station is 30° and the angle at the second tracking station is 70° ?

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$$h = \frac{s}{\cot \alpha - \cot \beta}$$

$$h = \frac{12.95}{\cot 30 - \cot 70}$$

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$$h = \frac{12.95}{\cot 30 - \cot 70}$$

$$h = \frac{12.95}{1.7320 - 0.3640}$$

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$$h = \frac{12.95}{1.3680}$$

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$$h = \frac{12.95}{1.3680}$$

$$h = 9.4664$$

The aircraft's height is 9.4664 miles.

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$$h = \frac{12.95}{1.3680}$$

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The aircraft's height is 9.4664 miles. To convert this to feet, we know that one mile equals 5,280 feet.

$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{9.4664}{h}$$

$$h = 9.4664 \times 5{,}280$$

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$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{9.4664}{h}$$

$$h = 9.4664 \times 5{,}280$$
$$h = 49{,}982.46 \text{ feet}$$

The aircraft is 49,982 feet above the ground.

Height of an Aircraft in Flight

Exercise 1:

Two tracking stations 20.6 miles apart observe an aircraft flying over them. How high, ***in feet,*** is the aircraft above the ground if the plane's angle of inclination at the first station is 20° and the angle at the second tracking station is 60° ?

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$$h = \frac{s}{\cot \alpha - \cot \beta}$$

$$h = \frac{20.6}{\cot 20 - \cot 60}$$

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$$h = \frac{20.6}{\cot 20 - \cot 60}$$

$$h = \frac{20.6}{2.7475 - 0.5774}$$

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$$h = \frac{20.6}{2.1701}$$

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$$h = \frac{20.6}{2.1701}$$

$$h = 9.4926$$

The aircraft's height is 9.4926 miles.

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The aircraft's height is 9.4926 miles. To convert this to feet, we know that one mile equals 5,280 feet.

$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{9.4926}{h}$$

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$$h = 9.4926 \times 5{,}280$$

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The aircraft's height is 9.4926 miles. To convert this to feet, we know that one mile equals 5,280 feet.

$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{9.4926}{h}$$

$h = 9.4926 \times 5{,}280$
$h = 50{,}120$ feet

The aircraft is 50,120 feet above the ground.

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The F-106 all-weather interceptor was developed from the Convair F-102 Delta Dagger. Originally designated F-102B, it was redesignated F-106 because it had extensive structural changes and a more powerful engine. The first F-106A flew on December 26, 1956, and deliveries to the Air Force began in July 1959. Production ended in late 1960 after 277 F-106As and 63 F-106Bs had been built. The F-106 used a Hughes MA-1 electronic guidance and fire control system. After takeoff, the MA-1 can be given control of the aircraft to fly it to the proper altitude and attack position. Then it can fire the Genie and Falcon missiles, break off the attack run and return the aircraft to the vicinity of its base. The pilot takes control again for the landing.
Ceiling: 53,000 ft.

Height of an Aircraft in Flight

Exercise 2:
An aircraft is flying at 10,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°?

$$h = \frac{s}{\cot \alpha - \cot \beta}$$

$$10{,}000 = \frac{s}{\cot 10 - \cot 80}$$

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$$h = \frac{s}{\cot \alpha - \cot \beta}$$

$$10{,}000 = \frac{s}{\cot 10 - \cot 80}$$

$$10{,}000 = \frac{s}{5.6713 - 0.1763}$$

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$$10{,}000 = \frac{s}{5.6713 - 0.1763}$$

$$10{,}000 = \frac{s}{5.495}$$

Height of an Aircraft in Flight

Exercise 2:

An aircraft is flying at 10,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°?

$$5.4950 \times 10{,}000 \text{ ft} = \frac{s}{5.495} \times 5.4950$$

Height of an Aircraft in Flight

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An aircraft is flying at 10,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°?

$$5.4950 \times 10{,}000 \text{ ft} = \frac{s}{5.495} \times 5.4950$$

54,950 ft = s The distance is 54,950 ft.

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An aircraft is flying at 10,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°?

$$5.4950 \times 10{,}000 \text{ ft} = \frac{s}{5.495} \times 5.4950$$

$$54{,}950 \text{ ft} = s$$

The distance is 54,950 ft.

One mile equals 5,280 ft.

$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{s}{54{,}950 \text{ feet}}$$

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An aircraft is flying at 10,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°?

$$5.4950 \times 10{,}000 \text{ ft} = \frac{s}{5.495} \times 5.4950$$

$$54{,}950 \text{ ft} = s$$

The distance is 54,950 ft.

One mile equals 5,280 ft.

$$\frac{1 \text{ mile}}{5{,}280 \text{ feet}} = \frac{s}{54{,}950 \text{ feet}}$$

$$s = \frac{54{,}950}{5{,}280}$$

$$s = 10.4017 \text{ miles}$$

The tracking stations are 10.4017 miles apart.

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The French-built Nieuport 28 became the first fighter airplane flown in combat by pilots of the American Expeditionary Force (AEF) in World War I. The lightly built Nieuport 28 developed a reputation for shedding its upper wing fabric in a dive, and by the spring of 1918, many considered the Nieuport 28 obsolete. Even so, American pilots maintained a favorable ratio of victories to losses with it. Many American aces of WWI, including 26-victory ace Capt. Eddie Rickenbacker, flew the Nieuport at one time or another in their careers. It had a ceiling of 17,000 ft.

More Resources

Additional Resources
are available online at
www.nationalmuseum.af.mil/education/teacher/index.asp