NATIONAL MUSEUM OF THE UNITED STATES AIR FORCE

MATHEMATICS OF FLIGHT: CROSSWINDS

Students will have a basic understanding of math applications used in flight. This includes the effects of crosswinds on aircraft course and direction. Students will solve a series of problems. (One in a series)

LESSON PLAN

Grade Level: High School-Trigonometry

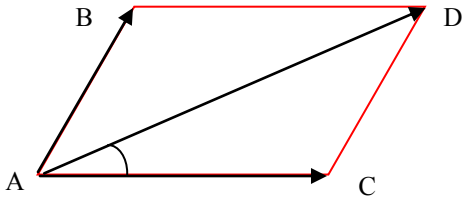
Common Core State Standards for Mathematics:
Trigonometric Functions: Model periodic phenomena with trigonometric functions, prove and apply trigonometric identities

Technology Content Standards (from STL):
Technology and Society

Lesson Objectives
The students will:
- Be introduced to formulas used in flight related to navigation and aircraft performance.
- Learn to calculate the effects of crosswinds on aircraft course and direction.

Goal
In this lesson, students will gain an understanding of common calculations performed by flight personnel.

Crosswinds
Wind often causes an aircraft to drift from its heading or direction. Pilots must calculate the effect the wind will have on the aircraft so they can remain on course.



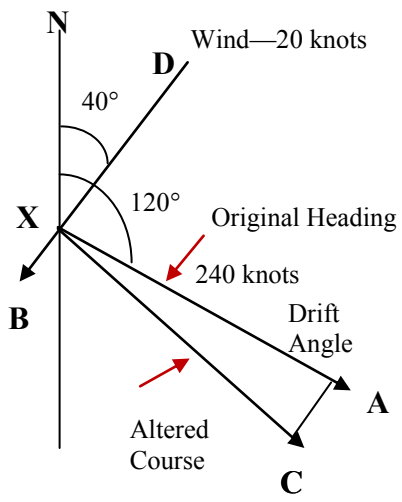
In still air, an aircraft would travel due east along $\overrightarrow{AC}$. With a crosswind blowing un the direction of $\overrightarrow{AB}$, the aircraft actually travels in the direction of $\overrightarrow{AD}$.
$\overrightarrow{AD}$ represents the course of the aircraft. $\angle CAD$ is called the drift angle.

Materials Required:

- Paper
- Pencil or pen
- Trigonometric Tables
- Calculator (optional)

Example:
Find the course, the speed of the wind and the drift angle of an aircraft headed at 120° when flying at 240 knots in still air if there is a crosswind of 20 knots blowing from the direction 40°.

Solution:
Draw a diagram to illustrate the problem.



∠AXC is the drift angle.
∠AXD + ∠AXB = 180°
(120 - 40) + ∠AXB = 180°
80 + ∠AXB = 180°
80 + ∠AXB - 80 = 180° - 80
∠AXB = 100°

Using alternate interior angles:
∠CAX = ∠AXD
∠AXD = 180° − ∠AXB
∠AXD = 180° − 100°
∠AXD = 80°
∠CAX = 80°

Using ΔXAC and the Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\Theta$

$\overrightarrow{XC}^2 = \overrightarrow{AC}^2 + \overrightarrow{XA}^2 - 2\overrightarrow{AC}\ (\overrightarrow{XA}\cos A)$

$\overrightarrow{XC}^2 = 20^2 + 240^2 - (2 \times 20)(240 \cos 80°)$

$\overrightarrow{XC}^2 = 400 + 57{,}600 - 9{,}600\ (0.1736)$

$\overrightarrow{XC}^2 = 400 + 57{,}600 - 1667$

$\overrightarrow{XC}^2 = 56{,}333$

$\overrightarrow{XC} = 237.35$

Using the Law of Sines:

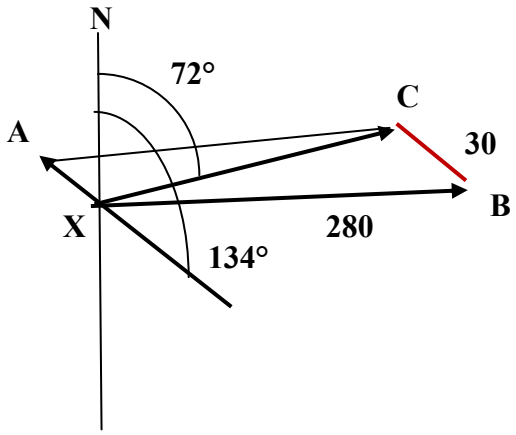
$$\frac{\sin\angle AXC}{AC} = \frac{\sin\angle CAX}{XC}$$

$$\frac{\sin\angle AXC}{20} = \frac{\sin 80°}{237.35} = \frac{0.9848}{237.35}$$

$$\sin\angle AXC = \frac{20\ (.09848)}{237.35} = 0.0830$$

$\angle AXC = 4.76°$

Exercise 1:
A pilot is to fly on course 72° in a wind blowing 30 knots from a direction of 134°. If the aircraft's speed is 280 knots, in what direction must the pilot head the aircraft and what will be the speed of the aircraft in the wind?



$\angle BCX = \angle AXC = 180° — (134° — 72°) = 118°$

$$\frac{\sin\angle BXC}{30} = \frac{\sin\angle BCX}{280}$$

$$\frac{\sin\angle BXC}{30} = \frac{\sin 118°}{280} = \frac{0.8829}{280}$$

$$\sin\angle BXC = \frac{30\,(0.8829)}{280} = 0.0946$$

$\angle BXC = 5.43°$

The aircraft heading must be $72° + 5.4° = 77.4°$

To find the speed in the wind,

$\angle XBC = 180° - \angle BCX - \angle BXC$

$\angle XBC = 180° - 118° - 5.43°$

$\angle XBC = 56.57°$

$$\frac{\overrightarrow{XC}}{\sin \angle XBC} = \frac{280}{\sin \angle BCX}$$

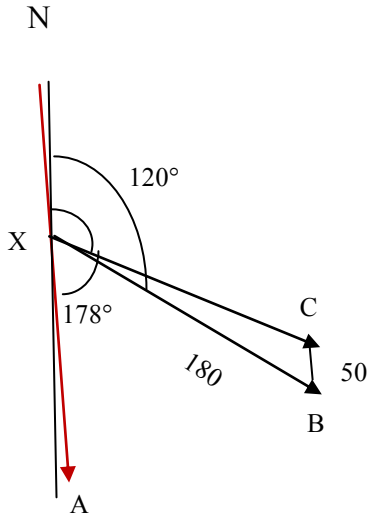
$$\frac{\overrightarrow{XC}}{\sin 56.57°} = \frac{280}{\sin 118°}$$

$$\overrightarrow{XC} = \frac{280 (\sin 56.57°)}{\sin 118°}$$

$$\overrightarrow{XC} = \frac{280 (0.8346)}{0.8829}$$

$$\overrightarrow{XC} = 264.66 \text{ knots}$$

Exercise 2:
An aircraft is headed in direction 120° with a speed of 180 knots in still air. The wind is blowing from 178°at 50 knots. What will be the course of the aircraft? What will be the speed of the aircraft in the wind?
(Round answer to the nearest hundredth.)



Using alternate interior angles:
$\angle CBX = \angle AXB = 178° - 120° = 58°$

Using $\angle BXC$ and the Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos\Theta$

$\overrightarrow{XC}^2 = \overrightarrow{BC}^2 + \overrightarrow{XB}^2 - 2\overrightarrow{BC}\ (\overrightarrow{XB} \cos \angle CBX)$

$\overrightarrow{XC}^2 = 50^2 + 180^2 - (2 \times 50)(180 \cos 58°)$

$\overrightarrow{XC}^2 = 2{,}500 + 32{,}400 - 18{,}000\ (0.5299)$

$\overrightarrow{XC}^2 = 2{,}500 + 32{,}400 - 9{,}538.20$

$\overrightarrow{XC}^2 = 25{,}361.80$

$XC = 159.25$

The aircraft will travel at 159.25 knots.

Using the Law of Sines:

$$\frac{\sin\angle AXC}{AC} = \frac{\sin\angle CAX}{XC}$$

$$\frac{\sin\angle AXC}{20} = \frac{\sin 80°}{237.3} = \frac{0.9848}{237.3}$$

$$\sin\angle AXC = \frac{20\,(.09848)}{237.3} = 0.0830$$

$\angle AXC = 4.76°$

See student worksheet and presentation.

Resources:

National Museum of the United States Air Force

Belcher, Diana. *Education in Flight: A Teacher's Guide to the Mathematics of Flight.* Department of the Air Force, 2007.

NATIONAL MUSEUM OF THE UNITED STATES AIR FORCE

MATHEMATICS OF FLIGHT: CROSSWINDS

STUDENT WORKSHEET *NAME:*_____

Exercise 1:
A pilot is to fly on course 72° in a wind blowing 30 knots from a direction of 134°. If the aircraft's speed is 280 knots, in what direction must the pilot head the aircraft and what will be the speed of the aircraft in the wind? (Round answer to the nearest hundredth.)

MATHEMATICS OF FLIGHT: CROSSWINDS—CONTINUED

STUDENT WORKSHEET

Exercise 2:
An aircraft is headed in direction 120° with a speed of 180 knots in still air. The wind is blowing from 178°at 50 knots. What will be the course of the aircraft? What will be the speed of the aircraft in the wind? (Round answer to the nearest hundredth.)