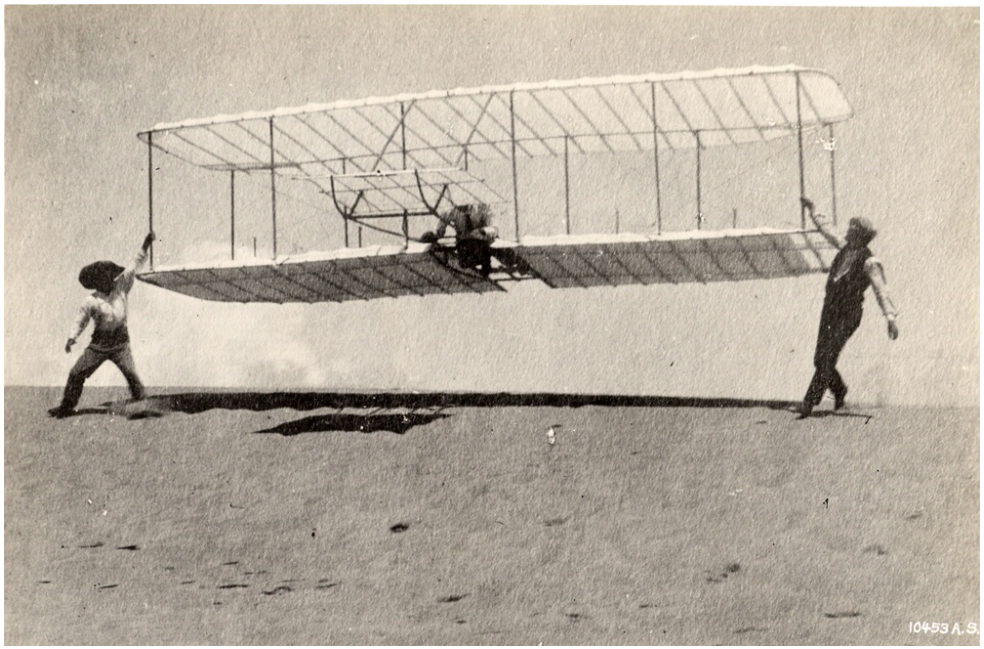
Mathematics of Flight

Glide Slope II

Glide Slope II

A glider is a special kind of aircraft that has no engine. The Wright brothers perfected the design of the first airplane and gained piloting experience through a series of glider flights from 1900 to 1903.



With Wilbur Wright at the controls, Dan Tate, left, and Edward C. Huffaker launch the 1901 Wright Glider at Kitty Hawk, N.C.

Credit: *Library of Congress, courtesy National Air and Space Museum, Smithsonian Institution*

Glide Slope II

During World War II, gliders such as the WACO CG-4 were towed aloft by C-47 and C-46 aircraft then cut free to glide over many miles.

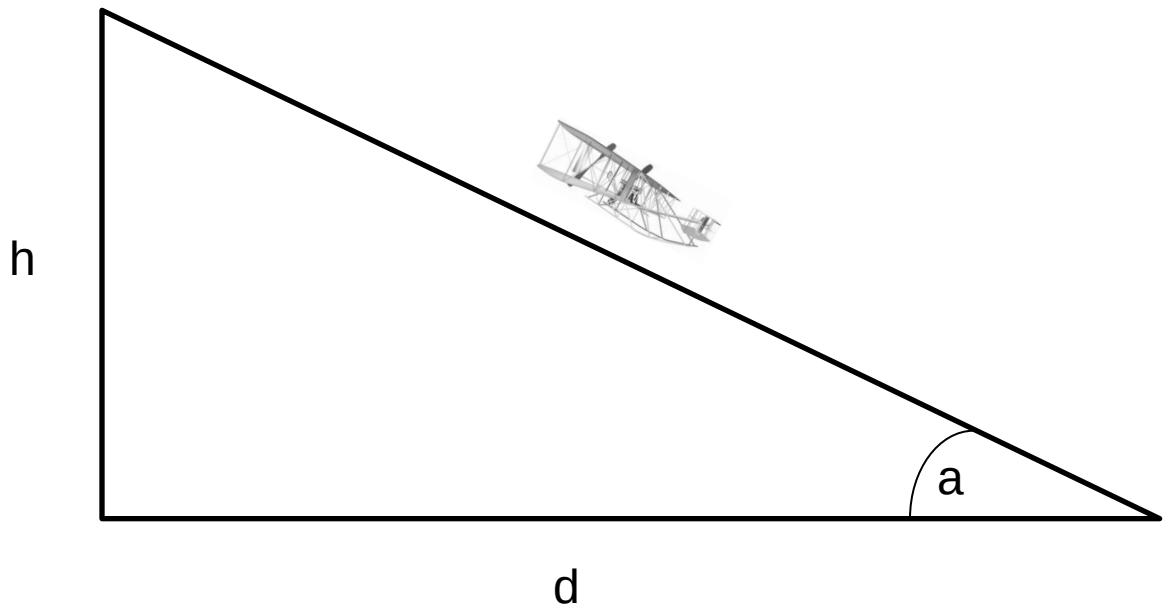
Glide Slope II

If a glider is in a **steady** (constant velocity and no acceleration) descent, it loses altitude as it travels. The glider's flight path is a simple straight line, shown in the figure above. The flight path intersects the ground at an angle **a** called the glide angle. If we know the distance flown and the altitude change, the glide angle can be calculated using trigonometry.

The tangent **tan** of the glide angle **a** is equal to the change in height **h** divided by the distance flown **d**:

$$\tan(a) = h / d$$

Glide Slope II



d = horizontal distance flown
h = change in height
a = glide angle

From trigonometry:

$$\tan(a) = \frac{h}{d}$$

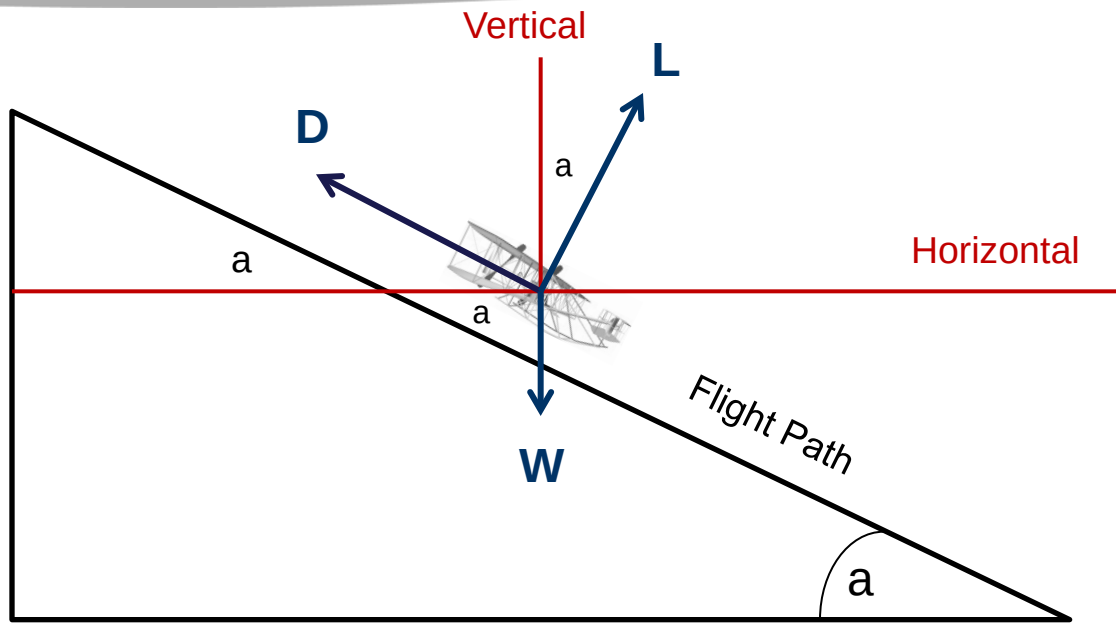
Glide Slope II

There are three forces acting on the glider; lift, weight, and drag. The weight of the glider is given by the symbol "**W**" and is directed vertical, toward the center of the earth. The weight is then perpendicular to the horizontal red line drawn parallel to the ground and through the center of gravity.

The drag of the glider is designated by "**D**" and acts along the flight path opposing the motion.

Lift, designated "**L**," acts perpendicular to the flight path. Using some geometry theorems on angles, perpendicular lines, and parallel lines, we see the glide angle "**a**" also defines the angle between the lift and the vertical, and between the drag and the horizontal.

Glide Slope II



L = Lift
D = Drag
W = Weight
a = glide angle

Vertical Equation: $L \cos(a) + D \sin(a) = W$

Horizontal Equation: $L \sin(a) = D \cos(a)$

$$\frac{\sin(a)}{\cos(a)} = \tan(a) = \frac{D}{L}$$

*Assuming velocity is constant

Glide Slope II

Assuming that the forces are balanced (no acceleration of the glider), we can write two vector component equations for the forces.

In the vertical direction, the weight **(W)** is equal to the lift **(L)** times the cosine (cos) of the glide angle **(a)** plus the drag **(D)** times the sine (sin) of the glide angle.

$$L * \cos(a) + D * \sin(a) = W$$

In the horizontal direction, the lift **(L)** times the sine (sin) of the glide angle **(a)** equals the drag **(D)** times the cosine (cos) of the glide angle.

$$L * \sin(a) = D * \cos(a)$$

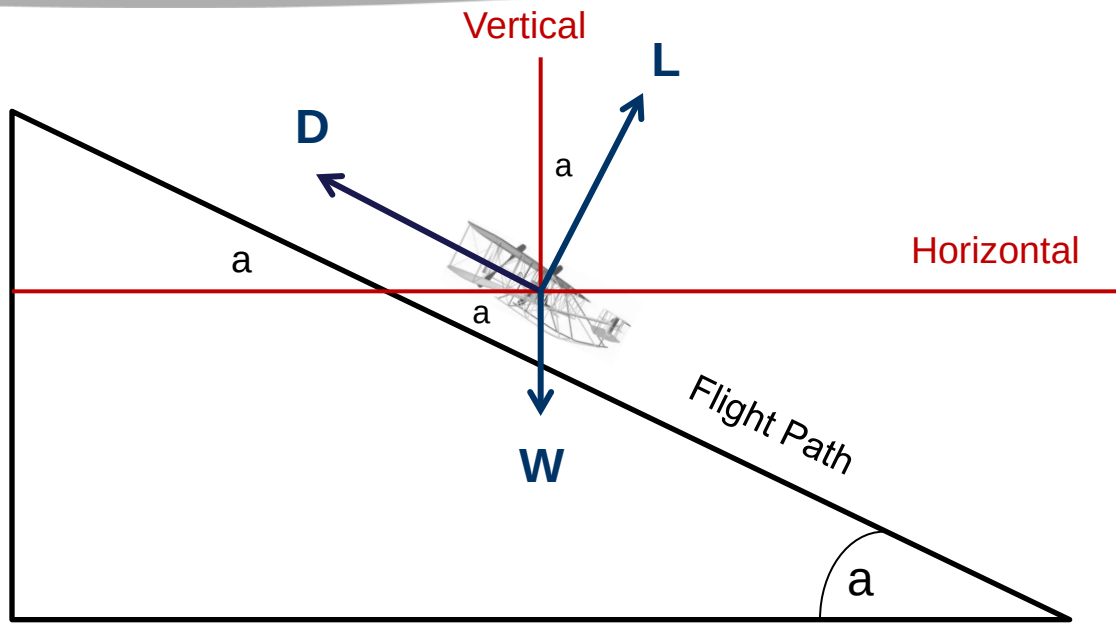
Glide Slope II

Lift = pressure factor x velocity squared x wing area x lift factor

Lift coefficient “cl” is the ratio of the object’s lift to the drag of the perpendicular flat plate with equal area.

Smeaton’s coefficient “k” is the drag of a 1 square foot flat plate at 1 mile per hour. 1900 accepted value =.005, modern accepted value = .00326

Glide Slope II



cl = Lift coefficient

cd = Drag coefficient

$$\frac{D}{L} = \frac{cd\ k\ V^2\ A}{cl\ k\ v^2\ A} = \frac{cd}{cl} = \tan(a)$$

Glide Slope II

If we use algebra to re-arrange the horizontal force equation we find that the drag divided by the lift is equal to the sine of the glide angle divided by the cosine of the glide angle. This ratio of trigonometric functions is equal to the tangent of the angle.

$$D / L = \sin(a) / \cos(a) = \tan(a)$$

We can use the drag equation and the lift equation to relate the glide angle to the drag coefficient **(cd)** and lift coefficient **(cl)** that the Wrights measured in their wind tunnel tests.

$$D / L = cd / cl = \tan(a)$$

Glide Slope II

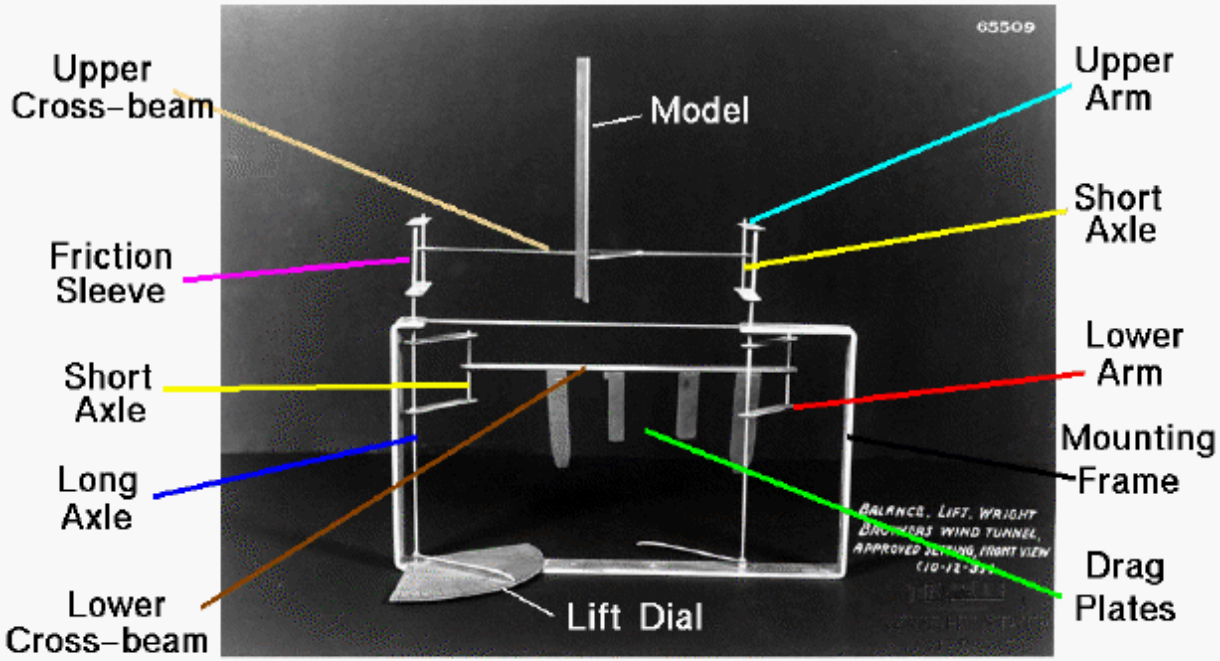
Wright Brothers
1901 Wind Tunnel

This is a replica of the wind tunnel designed and built by the Wright Brothers in the fall of 1901 to test airfoil designs. The blower fan, driven by an overhead belt, produced a 25 to 35 mph wind for testing the lift of various planes and curved surfaces. Aerodynamic tables derived from these tests were vital to the successful design of the Wright 1903 Kitty Hawk airplane. Inside the tunnel is a model of a Wright lift balance used to measure the lift of a test surface. The wind tunnel replica was constructed under the personal supervision of Orville Wright prior to World War II.

Glide Slope II

Wright 1901 Lift Balance Design

Glenn Research Center



View Looking Downstream

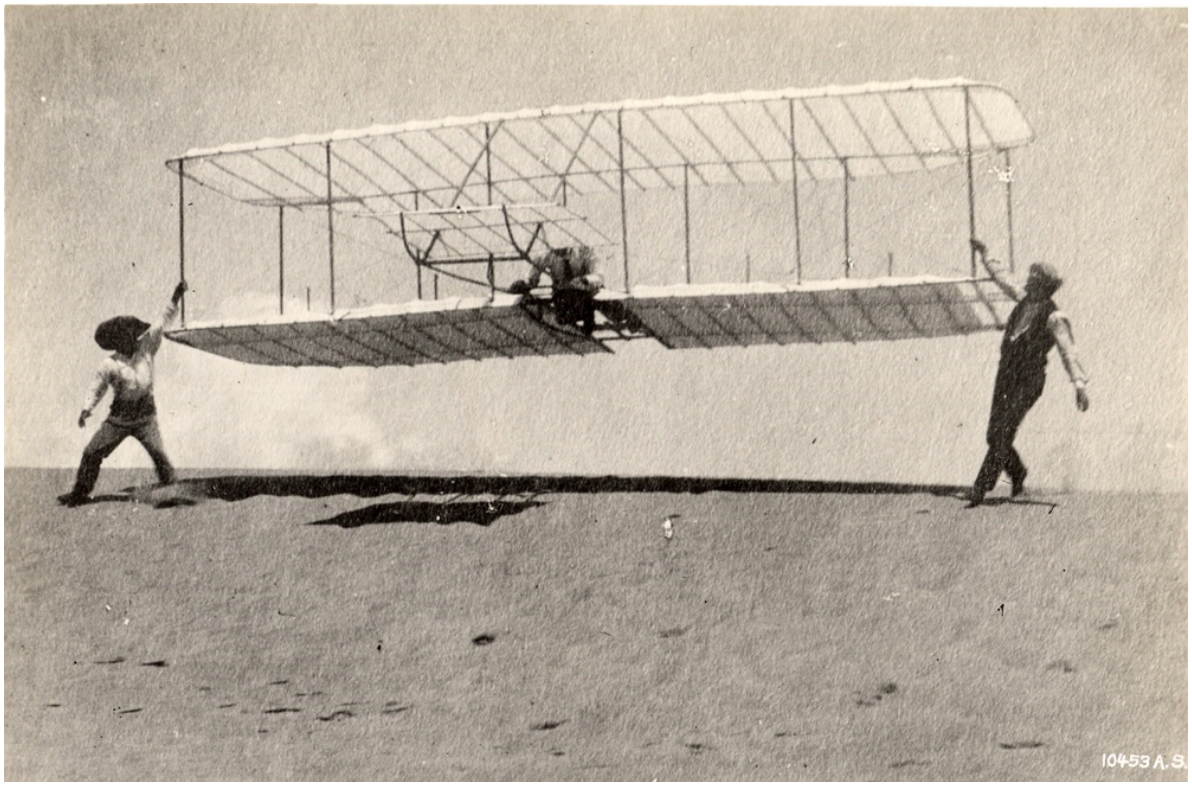
Glide Slope II

During the operation of the drag balance the brothers made measurements of the effects of wing design on glide angle through the drag to lift ratio.

The inverse of the drag to lift ratio is the L/D ratio which is an efficiency factor for aircraft design.

The higher the L/D, the lower the glide angle, and the greater the distance that a glider can travel across the ground for a given change in height.

Glide Slope II



With Wilbur Wright at the controls, Dan Tate, left, and Edward C. Huffaker launch the 1901 Wright Glider at Kitty Hawk, N.C.

Credit: *Library of Congress, courtesy National Air and Space Museum, Smithsonian Institution*

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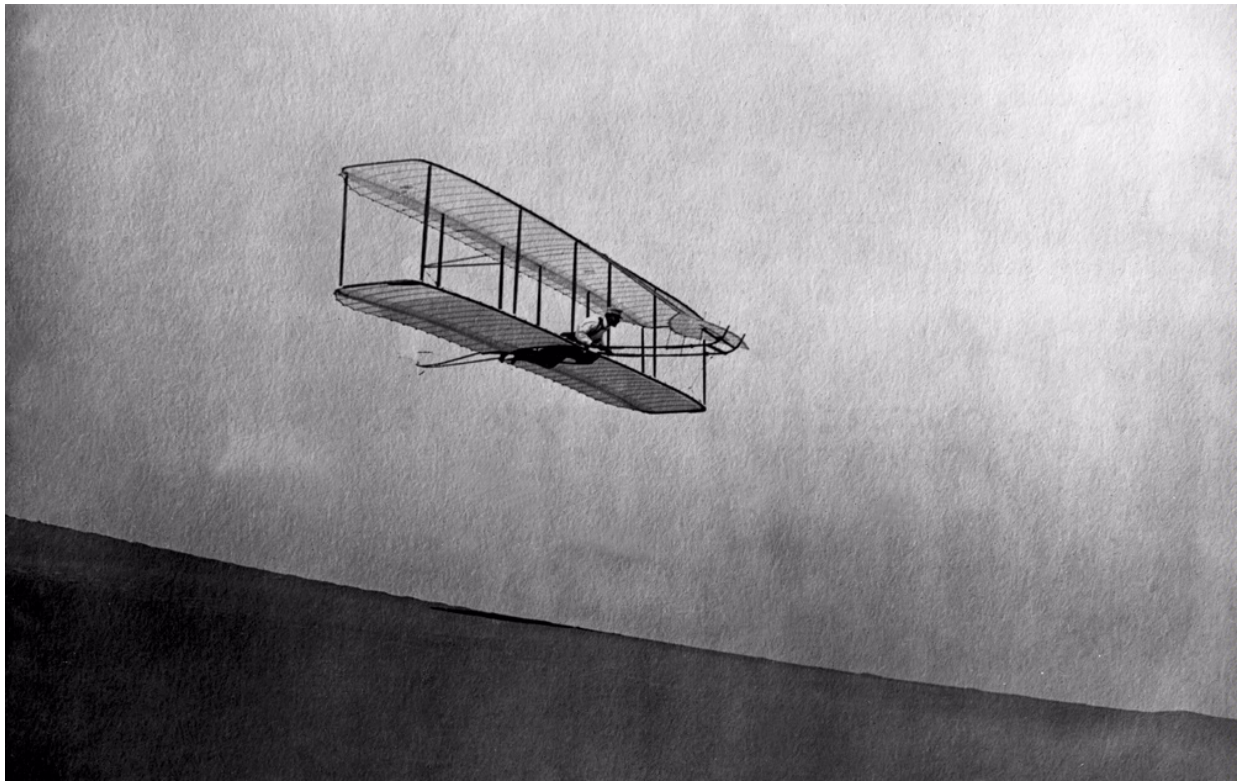


Wilbur Wright gliding in 1902. The Wrights added a vertical tail to their glider to deal with the lateral control problems experienced in 1901. The more graceful appearance of the 1902 machine over the previous gliders is evident in this picture.

Credit: *National Air and Space Museum, Smithsonian Institution*

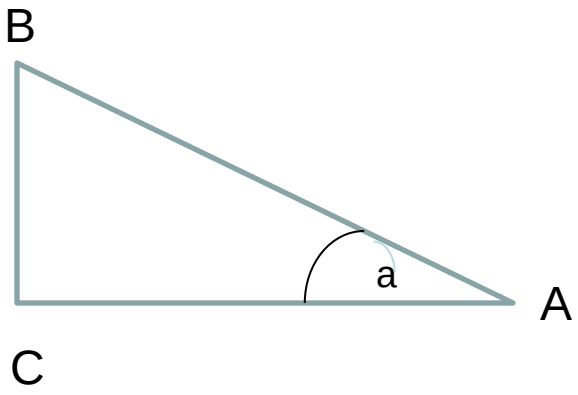
Glide Slope II

Compare the 1901 and 1902 Wright gliders

Glider	Weight	Distance	Glide Angle	Height
1901	98 lbs	300 feet	9°	46.9 feet
1902	117 lbs	500 feet	7°	?

For the 1901 Wright Glider:

sin(a) = BC/AB
AB = distance flown = 300 ft
a = glide angle
BC = height



sin(a) = BC/ 300 ft
.1564 = BC/300 ft
46.9 ft = BC

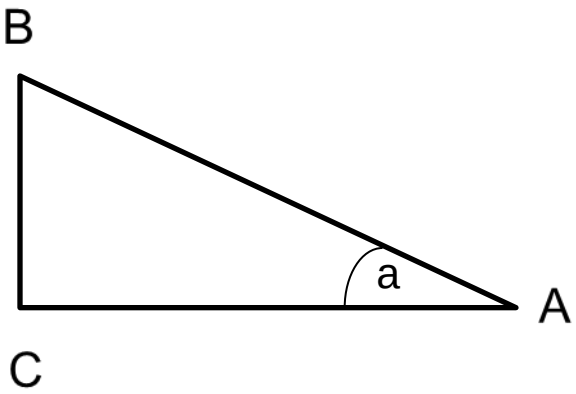
Glide Slope II

Compare the 1901 and 1902 Wright gliders

Glider	Weight	Distance	Glide Angle	Height
1901	98 lbs	300 feet	9°	46.9 feet
1902	117 lbs	500 feet	7°	?

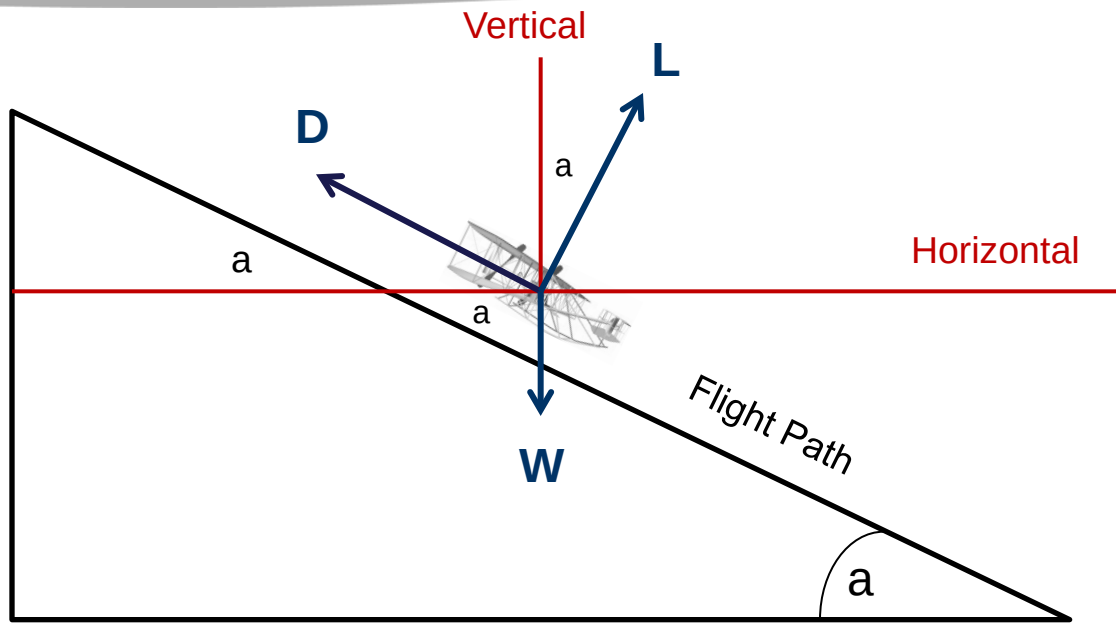
For the 1902 Wright Glider:

sin(a) = BC/AB
AB = distance flown = 500 ft
a = glide angle
BC = height



sin(a) = BC/ 500 ft
.1219 = BC/500 ft
60.9 ft = BC

Glide Slope II



L = Lift
D = Drag
W = Weight
a = glide angle

Vertical Equation: L cos(a) + D sin(a) = W
Horizontal Equation: L sin(a) = D cos(a)

$$\frac{\sin(a)}{\cos(a)} = \tan(a) = \frac{D}{L}$$

Glide Slope II

For the 1901 glider:

$L * \cos(a) + D * \sin(a) = W$

$L \times .9877 + D \times .1564 = 98$

$.9877L + .1564D = 98$

The lift times the sine of the glide angle is equal to the drag times the cosine of the glide angle.

$L * \sin(a) = D * \cos(a)$

$L * .1564 = D * .9877$

$$\frac{.1564L}{.9877} = \frac{.9877D}{.9877}$$

$$\frac{.1564L}{.9877} = D$$

Now, substitute for D, $.9877L + \frac{.1564\,(.1564)}{.9877} = 98$

More Resources

Additional Resources
are available online at
www.nationalmuseum.af.mil/education/teacher/index.asp