Mathematics of Flight: Crosswinds

Students will practice using trigonometric functions by learning about the effects of crosswinds on aircraft flight. Vector quantities and the practice of vector addition will be highlighted.

LESSON PLAN

Lesson Objectives

The students will:
- Be introduced to formulas used in flight, related to navigation and aircraft performance.
- Learn to calculate effects of crosswinds on aircraft course and direction.
- Learn applications of the law of sines, law of cosines, and vector additions.

Goal

In this lesson, students should learn to use trigonometry to determine quantities such as drift angle, drift velocity and others. Students will also demonstrate their ability to recreate a diagram from worded examples.

Background

Crosswinds are any winds that have an effect on an aircraft's flight path. There are special cases such as headwinds, where the wind acts opposite to the planes direction. Other special cases include a tailwind, where the plane and wind are acting in the same direction. As you may suspect the relative velocity of the aircraft increases when there is a tail wind, and decreases when there is a head wind. In these simple one dimensional cases, the relative velocity is easily calculated. Often times, the world is not so ideal. Suppose you were trying to fly due north, but you had a pesky wind blowing you due east – without prior factoring that into your flight trajectory, the wind would blow you a bit to the east.

Lastly, before getting started, velocity is a vector quantity, meaning that it is a quantity with a **magnitude** and a **direction**. For example, an airplane could be travelling 150 knots (a magnitude) due east ($\overrightarrow{AC}$ in the figure) and wind would blow a cross wind at 50 knots northeast ($\overrightarrow{AB}$ in the figure). The **trajectory** is given by $\overrightarrow{AD}$ and the **drift angle** is the measure of deviation from the original flight course ($\angle DAC$).

Grade Level:	**9 – 12**

Ohio Learning Standards/Science (2018)
Expectation of Learning
Nature of Science
Physical Science
PS.FM.1 Motion
Physics
P.F.6 Vectors in two dimensions
Ohio Learning Standards/Mathematics (2017)
Vector and Matrix Quantities
N.VM.1 Recognize Vector quantities
N.VM.3 Solve problems using vectors
N.VM.4 Add and subtract vectors
Geometry
G.CO.1 Know geometric terminology
G.CO.7 Congruence of triangles given sides and angles
G.CO.8 Criteria for triangles and congruence
G.CO.9 Prove and apply theorems (line/angles)
G.CO.10 Prove and apply theorems (triangles)
G.CO.11 Prove and apply theorems (parallelograms)
G.CO.12 Make geometric constructions
G.SRT.4 Prove and apply theorems (triangles)
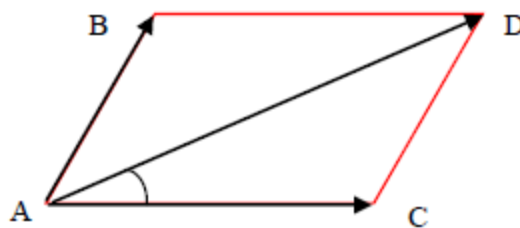G.SRT.5 Use congruence and similarity to solve
G.SRT.10 Use the law of sine/cosines to solve
Algebra
A.REI.6 Solve rational and radical equation of a single variable

Materials Required:
Paper, writing utensil, graphing calculator, protractor, graph paper

Simple Motion (1D Case)
If there is a headwind or a tailwind it is easy to demonstrate the resultant velocity.
Example: An aircraft is initially cruising at 240 knots forward, when suddenly, a headwind starts blowing at 15 knots in the direction opposite the aircraft. What is the resultant velocity of the aircraft?

Solution: Visually:



5 knots   240 knots

Resultant Velocity
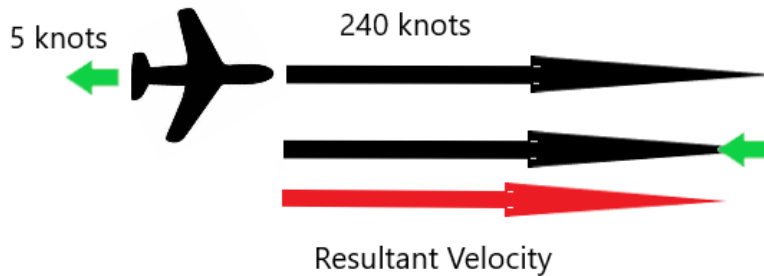
Since the wind and the plane's velocity can be given by vectors, we line the vectors "tip to tail", which is vector addition. After the vectors are added, the resulting vector is the observed velocity of the aircraft. In this example, the resultant velocity is 240-5 knots, or **235 knots.**
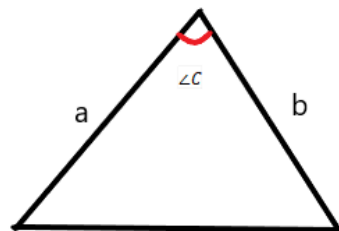
A More Realistic Case (2D Case)
When performing the following, we continue to add vectors from "tip to tail". There are a few equations that help when performing these calculations, being the law of sines and law of cosines, and other trigonometric identities.
The law of sines is illustrated below:
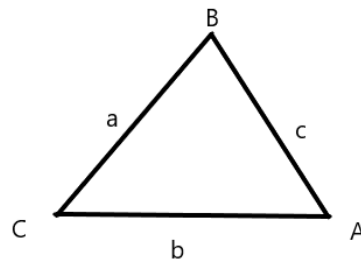
$$\frac{\sin(\angle A)}{a} = \frac{\sin(\angle B)}{b} = \frac{\sin(\angle C)}{c}$$

The law of cosines is illustrated below:

$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$

Example 2: A pilot is initially flying at 324 knots, 68.2° North of East. A cross wind then begins to blow at 56 knots 45° South of East. Find the drift angle, and what is the resultant velocity of the plane (that is, the speed and direction)?

Solution: We start by drawing a diagram and then connecting tip to tail and draw the vector we are trying to solve.
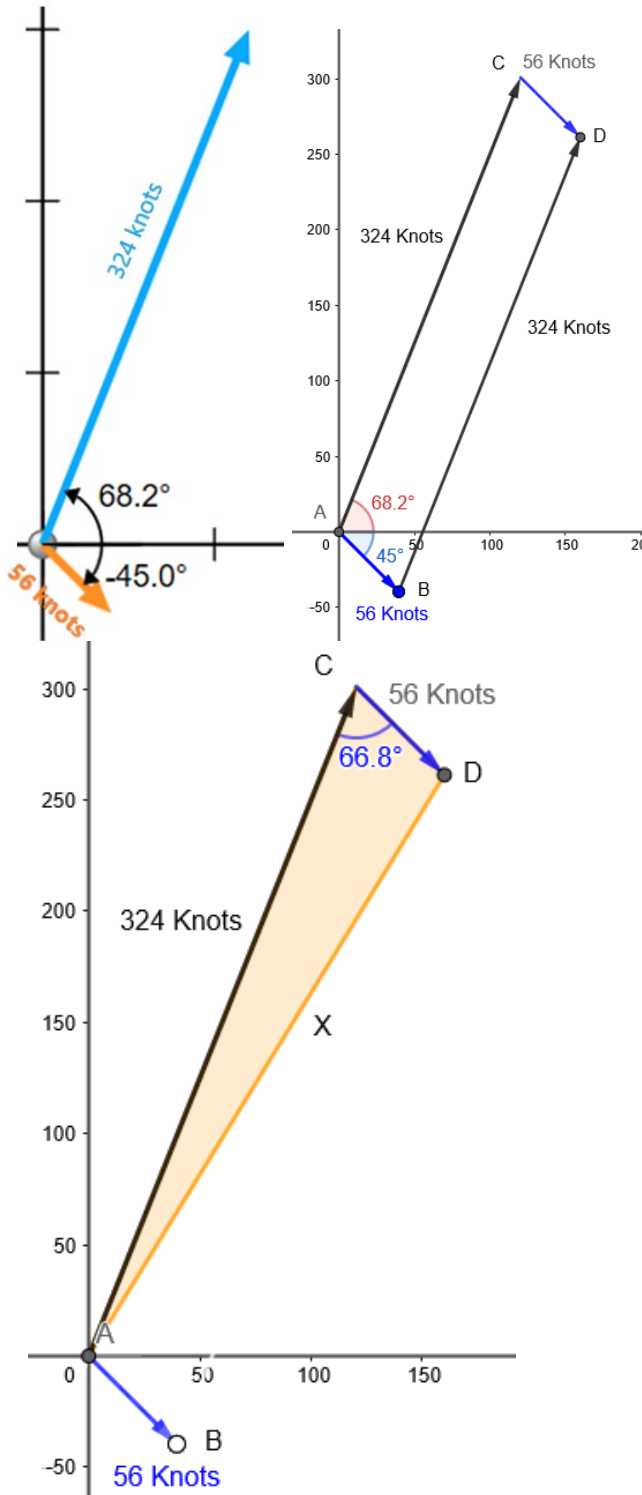
We notice here that the angle on opposite sides of the parallelogram must be equal to each other ($\angle A = \angle D$ and $\angle C = \angle B$). We also know that the sum of angles in a quadrilateral must add up to 360°.

$$360 = 2\angle A + 2\angle C$$

$$360° = 2(68.2° + 45°) + 2\angle C$$

$$\angle C = \frac{360° - 2(68.2° + 45°)}{2}$$

$$\angle C = 66.8°$$

Next, we draw the resultant vector to transform this into a trigonometry problem. We need to solve for the length of $\overrightarrow{AD}$ and $\angle DAC$ (the drift angle).

Given the two side lengths and the angle for the unknown side length, we are in the perfect position to use the law of cosines to solve for the side length, or magnitude of $\overrightarrow{AD}$, indicated as X in the aside figure.

$$X^2 = 324^2 + 56^2 - 2 * 324 * 56 * \cos(66.8°)$$

$$X^2 = 93816.64$$

$$X = 306.295 \; knots$$

Now that we have a corresponding side length with an interior angle (that is $\angle C$ to X), we may use the law of sines to determine the drift angle.

$$\frac{\sin(\angle A)}{56 \; Knots} = \frac{\sin(66.8°)}{306.295 \; knots}$$

$$\sin(\angle A) = \frac{56\sin(66.8°)}{306.295} \rightarrow \angle A = \sin^{-1}\left(\frac{56\sin(66.8°)}{306.295}\right)$$

$$\angle A = 9.67°$$

So the **drift angle is 9.67°,** and thus the trajectory of flight given by 68.2° − 9.67°= **58.53° N of E at 306.295 Knots**.

Example 3: A Pilot is attempting to plan their flight. They know that winds are blowing at a constant rate of 77 knots 30° South of West, and they want to travel due E at 200 knots. What bearing and speed should the pilot take to account for the crosswind?

Solution: Before we draw the picture, consider that the question is giving us the problem as follows – we are given the resultant vector ($\vec{R}$) that is a sum of the crosswind ($\vec{C}$) and the flight in still wind ($\vec{F}$), i.e $\vec{R} = \vec{C} + \vec{F}$

Thus, it should be noted that we need to solve for $\vec{F} = \vec{R} - \vec{C}$

We draw the provided vector quantities. Because we are going to subtract the crosswind, we 'flip' the vector so that it is now facing the opposite direction (see the image for clarity).
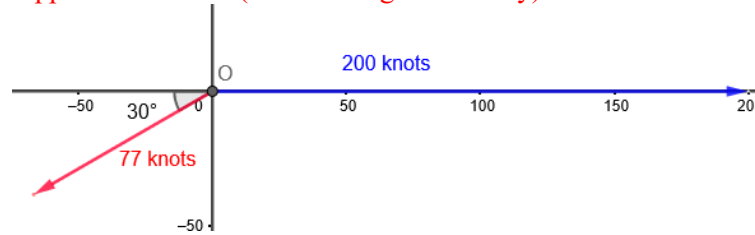
Now, we preform $\vec{F} = \vec{R} - \vec{C}$ which provides the following triangle to solve
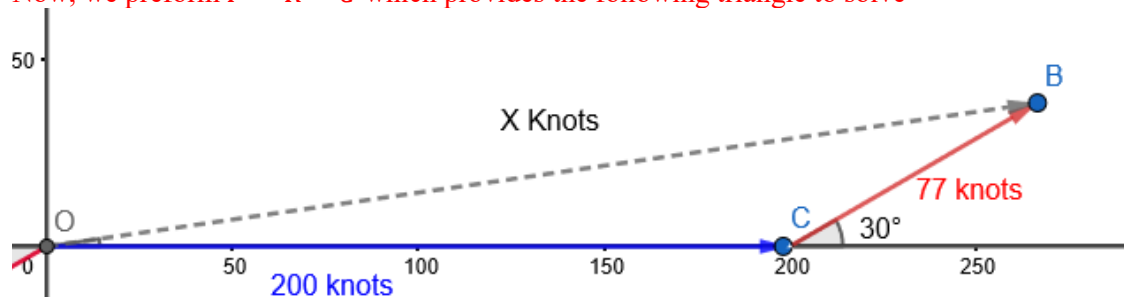
Once this is performed, it is easy to solve for $\angle C$.

$$\angle C = 180° - 30° = 150°$$

Then, it is possible to use the law of cosines again, to determine the length of the triangle, and hence, the speed of the aircraft.

$$X^2 = 200^2 + 77^2 - 2 * 200 * 77 \cos(150) = 72602.582$$

$$X = 269.449 \, knots$$

And resultantly, we now have enough information to use the law of sines to determine $\angle BOC$.

$$\frac{\sin(\angle BOC)}{77 \, Knots} = \frac{\sin(150°)}{269.449 \, knots}$$

$$\sin(\angle BOC) = \frac{77 \sin(150°)}{269.449} \rightarrow \angle A = \sin^{-1}\left(\frac{77 \sin(150°)}{269.449}\right) = 8.21°$$

So the pilot should fly at **269.449 Knots at 8.21° N of E** to compensate for the crosswind.

Name_____

Mathematics of Flight: Crosswinds

Round answers to the nearest hundredth. Sketch figures as necessary.

1) A pilot is initially flying at 180 knots, 25.2° South of East. A cross wind then begins to blow at 24 knots 6° East of South. Find the drift angle and the resultant velocity of the plane (that is, the speed and direction)?

2) A crow is initially flying at 34 knots, 15° West of North. A cross wind then begins to blow at 20 knots 45° East of north. What is the resultant velocity of the crow (that is, the speed and direction)?

3) A Pilot is attempting to plan their flight. They know that winds are blowing at a constant rate of 24 knots 30° North of East, and they want to travel due NE at 175 knots. What bearing and speed should the pilot take to account for the crosswind?

4) An aircraft is headed in the direction of 30° S of E with a speed of 180 knots in still air. The wind is blowing at 50 knots, 2° N of W. Find the drift angle and the resultant velocity of the plane?

Name_____KEY_____

Mathematics of Flight: Crosswinds

Round answers to the nearest hundredth. Sketch figures as necessary.

1) A pilot is initially flying at 180 knots, 25.2° South of East. A cross wind then begins to blow at 24 knots 6° East of South. Find the drift angle and the velocity of the plane (that is, the speed and direction)?

   Drift angle is 6.09°. The velocity in the wind is 193.52 knots, 31.29° S of E (58.71° E of S is equivalent)

2) A crow is initially flying at 34 knots, 15° West of North. A cross wind then begins to blow at 20 knots 45° East of north. What is the resultant velocity of the crow in the wind (that is, the speed and direction)?

   The velocity in the wind is 47.29 knots, 6.49° E of N (83.51° N of E is equivalent)

Name_____KEY_____

3) A Pilot is attempting to plan their flight. They know that winds are blowing at a constant rate of 24 knots 30° North of East, and they want to travel due NE at 175 knots. What bearing and speed should the pilot take to account for the crosswind?

The pilot should fly at 151.94 Knots at 42.66° E of N to compensate for the crosswind. (47.34° N of E is equivalent)

4) An aircraft is headed in the direction of 30° S of E with a speed of 180 knots in still air. The wind is blowing at 50 knots, 2° N of W. Find the drift angle and the resultant velocity of the plane?

Drift angle is 9.80°. The velocity in the wind is 137.87 knots, 39.80 ° S of E (50.20° E of S is equivalent)