Mathematics of Flight: Glide Slope II

Students will develop a basic understanding of math applications used in flight. This includes the glide slope or glide angle. Using trigonometry, students will solve a series of real world problems. (Two in a series of two).

LESSON PLAN

Lesson Objectives

The students will:

- Be introduced to formulas used in flight, related to navigation and aircraft performance.
- Learn to calculate the glide slope using trigonometry.
- Learn about the proportionality of the drag and lift coefficients, and how they are then related to the glide slope.

Goal

In this lesson, students should learn to use trigonometry to derive formulas to solve for desired quantities. Using this, in conjunction with algebraic manipulation, comfortability should be attained by practice. Students should also develop a physical understanding for drag and lift coefficients.

Background

A glider is a special kind of aircraft that has no engine. The Wright brothers perfected the design of the first airplane and gained piloting experience throughout a series of glider flights from 1900 to 1903. During World War II, gliders such as the WACO CG-4 were towed aloft by C-47 and C-46 aircraft then cut free to glide over many miles.

If a glider is in a steady (constant velocity, no acceleration) descent the forces on the plane can be considered equal. In a steady descent, the angle of the descent remains the same, and the forces acting on the aircraft can be assumed to be balanced. The three forces we consider are ***drag, lift*** and ***weight.*** The angle that the aircraft makes with the ground is the ***glide angle***. See the aside image for a simplified free body diagram.

The green arrow represents the force of drag, D.
The blue arrow represents the force of weight, W.
The red arrow represents that force of lift. L.
The angle indicates the glide angle (α, in degrees).

Grade Level: 9-12

Ohio Learning Standards/Science (2018)

Expectation of Learning

Nature of Science

Physical Science

PS.FM.2 Forces and Motion
PS.FM.3 Dynamics ($F_{net}=0$)

Physics

P.F.1 Newton's Laws applied
P.F.5 Air resistance and drag
P.F.6 Forces in two dimensions

Ohio Learning Standards/Mathematics (2017)

Vector and Matrix Quantities

N.VM.1 Recognize Vector quantities
N.VM.3 Solve problems using vectors
N.VM.4 Add and subtract vectors
N.VM.8 Multiply Matrices of apt dimensions
N.VM.11 Understand matrix multiplication of vectors as linear transformations

Geometry

G.CO.1 Know geometric terminology
G.CO.5 Understand transformations of figures
G.CO.10 Prove and apply theorems (triangles)

Algebra

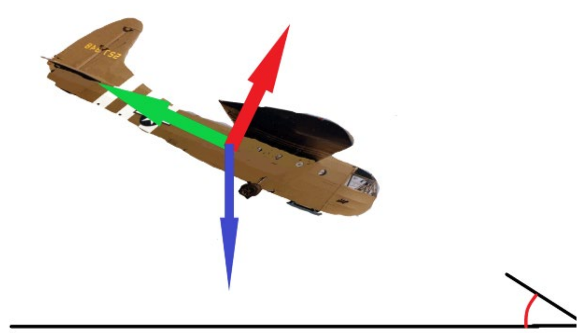
A.REI.5 Solve 2D systems of equations

Functions

F.TF.8 Prove Trig. Identities
F.TF.9 Use Trig. Identities to solve problems

Materials Required:

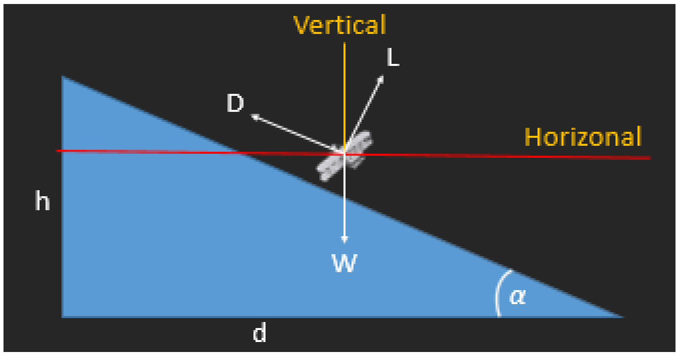
- Paper
- Writing Utensil
- Scientific or graphing calculator



Activity Summary

From the three forces which are acting on the glider, we have multiple approaches when trying to solve the problem. Each approach will yield the same answer, but in some ways, personal preference can dictate how one may find their way to the solution of these problems. Two methods are outlined below: 1) using the standard coordinate system 2) applying a transform to rotate into a shifted coordinate system.

Staying in the Standard Coordinate System



In the standard coordinate system, it is possible to decompose each of the vector forces into the respective x and y components. The designation is that the "Horizontal" text rests at 0°.

$$\vec{L} = \begin{bmatrix} L\ \cos(90-\alpha) \\ L\sin(90-\alpha) \end{bmatrix} \quad \vec{D} = \begin{bmatrix} D\cos(180-\alpha) \\ D\sin(180-\alpha) \end{bmatrix} \quad \vec{W} = \begin{bmatrix} W\cos(270) \\ W\sin(270) \end{bmatrix} = \begin{bmatrix} 0 \\ -W \end{bmatrix}$$

Since the forces assumed to be balanced, the sum of forces must be equal to zero in both the x and the y directions. That is:

$$\vec{L} + \vec{D} + \vec{W} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using some trigonometric identities, it is left to the reader to arrive at the following formulas.

$$L\cos(\alpha) + D\sin(\alpha) = W$$

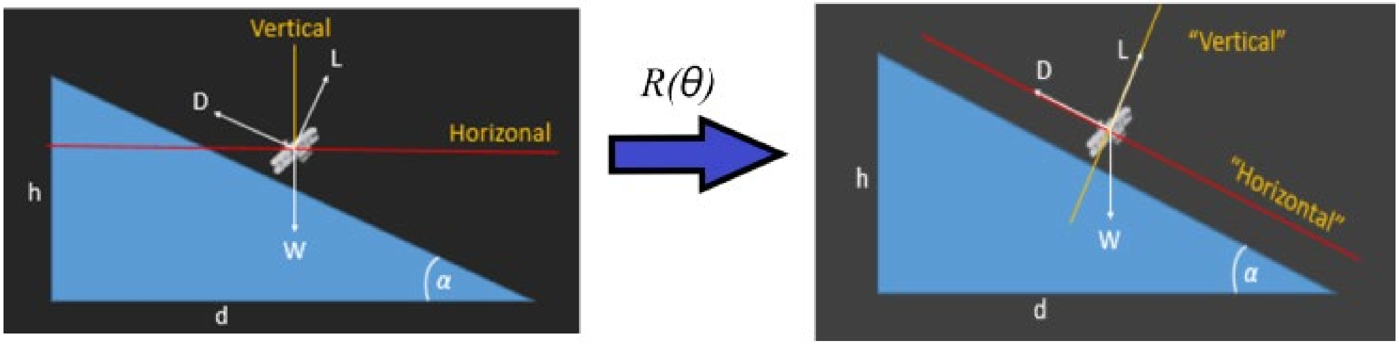
$$L\sin(\alpha) = D\cos(\alpha)$$

$$\frac{D}{L} = \tan(\alpha)$$

In these examples, the L, D, and W, without the arrow indicate the magnitude of the vector, and $\vec{L}, \vec{D}, \vec{W}$ indicate the vector for lift, drag, and weight respectively.

Solving Using a Rotated Coordinate System

In some systems, it may be easier to solve if we first apply a transformation to the system. Suppose we rotate the coordinate system by α degrees counterclockwise. This is illustrated by the figure on the next page.



See **Exercise 5** for a detailed example for applying this transformation. This transformation helps us with this example, as it simplifies the components. Using our modified coordinate system, we can determine the new horizontal and new vertical components of our vectors. Modifications indicated in red, due to the rotation operation).

$$\overrightarrow{L'} = \begin{bmatrix} L\cos(90-\alpha+\alpha) \\ L\sin(90-\alpha+\alpha) \end{bmatrix} = \begin{bmatrix} 0 \\ L \end{bmatrix} \quad \overrightarrow{D'} = \begin{bmatrix} D\cos(180-\alpha+\alpha) \\ D\sin(180-\alpha+\alpha) \end{bmatrix} = \begin{bmatrix} -D \\ 0 \end{bmatrix} \quad \overrightarrow{W'} = \begin{bmatrix} W\cos(270+\alpha) \\ W\sin(270+\alpha) \end{bmatrix} = \begin{bmatrix} W\sin(\alpha) \\ -W\cos(\alpha) \end{bmatrix}$$

These vectors, however, are in an alternate coordinate system (up is no longer up!) so care must be taken when indicating the direction of the force.
Since the forces assumed to be balanced, the sum of forces must be equal to zero in both the x and the y directions. That is:

$$\overrightarrow{L'} + \overrightarrow{D'} + \overrightarrow{W'} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using some trigonometric identities, it is left to the reader to arrive at the following formulas.

$$D = W\sin(\alpha)$$

$$L = W\cos(\alpha)$$

$$\frac{D}{L} = \tan(\alpha)$$

Extension to Lift and Drag Coefficients

We can determine an additional method to find the drag to lift ratio (or the tan(α) quantity) which is by determination of the drag and lift coefficients. There are specific formulas for each of these quantities. The lift coefficient, c_L, is unit-less and related to lift via the following equation.

$$Lift = \frac{1}{2} c_L * \rho * v^2 * A$$

In this equation, the lifting force is given in Newtons (N), ρ is the air density (in kg/m^3), v is the velocity of the aircraft (in m/s), and A is the area of the lifting body (e.g. wing, in m^2). Often times, the lifting coefficient is experimentally determined, and is used to factor many complications of flight.

In comparison, the equation for the drag coefficient, c_D, is very similar. The drag coefficient functions similarly to lift coefficient, where it acts as a simplification of complex forces to a single unit-less variable.

$$Drag = \frac{1}{2} c_D * \rho * v^2 * A$$

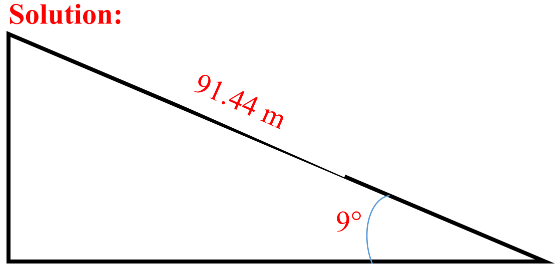
In this equation, the drag force is given in Newtons (N), ρ is the air density (in kg/m^3), v is the velocity of the aircraft (in m/s), and A is the area of the lifting body (e.g. wing, in m^2). Often times, the drag coefficient is experimentally determined.
If we then choose to determine the previous quantity of D/L, we can take our formulas as given and simplify.

$$\frac{Drag}{Lift} = \frac{\cancel{\frac{1}{2}} c_D \cancel{* \rho * v^2 * A}}{\cancel{\frac{1}{2}} c_L \cancel{* \rho * v^2 * A}} = \frac{c_D}{c_L} = \tan(\alpha)$$

The drag to lift ratio (D/L) and its inverse (L/D) help to indicate the efficiency of an aircraft. In general, the higher the L/D the lower the glide angle, and the greater the distance that a glider can travel across the ground for a given change of height.

Example 1: The 1901 Wright Glider has a mass of 44.44 kg, and during a test "flight" glided at an angle of 9° for a glide length of 91.44 m. Using this information draw the free body diagram, determine the height of the flight, $\vec{L}, \vec{D}, \vec{W}$, and the D/L ratio. (use g = 9.81 m/s^2). Use both methods of the standard and rotated coordinate system

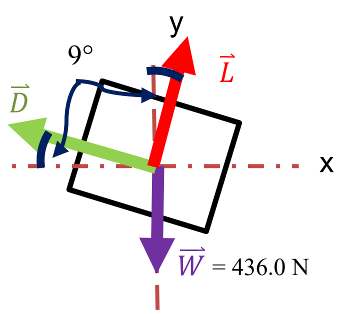
Solution:



Graphically, this is the information give that is relevant to determining the height of the glider. Using trigonometry, we notice:

$$\sin(9°) = \frac{h}{91.44\ m}$$

Thus, h = 91.44m *sin (9°) = 14.30m is the height that the glider descended from.



We also need to determine the force due to weight from the mass of the flyer. Given the mass of the glider is 44.44kg we determine the weight force to be 44.44 kg * 9.81 m/s^2 = 436.0 N downward.

Using the conventional coordinate axis, we have the following free body diagram. (See the left).
Since we have two unknowns, we can use a system of equations to solve this using the following formulas.

$$L\cos(9°) + D\sin(9°) = 436.0N$$

$$\frac{D}{L} = \tan(\alpha) \rightarrow D = L\tan(9°)$$

Thus by substitution

$$L\cos(9°) + L\tan(9°)\sin(9°) = 436.0\ N$$

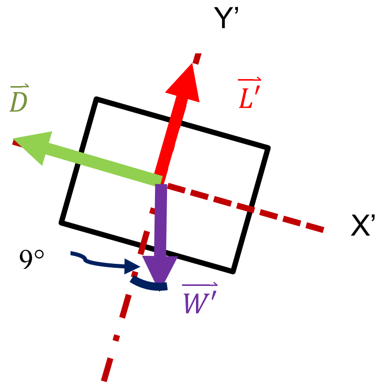
Rearranging and solving yields

$$L = \frac{436.0N}{\cos(9°) + \tan(9°)\sin(9°)} = 430.6\ N$$

And subsequently

$$D = 430.6\ N * \tan(9°) = 68.21\ N\ so\ {}^{D}/_{L} = 0.1584$$

$$\vec{L} = \begin{bmatrix} 430.6N\cos(81°) \\ 430.6N\sin(81°) \end{bmatrix} = \begin{bmatrix} 67.36N \\ 425.3N \end{bmatrix} \quad \vec{D} = \begin{bmatrix} 68.21N\cos(171°) \\ 68.21N\sin(171°) \end{bmatrix} = \begin{bmatrix} -67.36N \\ 10.7N \end{bmatrix} \quad \vec{W} = \begin{bmatrix} 0 \\ -436\ N \end{bmatrix}$$



Using the rotated coordinate system, the problem actually becomes simpler.
Given that we have the weight force of 436.0 N, we can use the following formulas to directly calculate L and D

$$D = W\sin(\alpha)\ \ and\ L = W\cos(\alpha)$$

Inputting the values from the problem statement …

$$D = 436.0N\sin(9°) = 68.21\ N$$

And

$$L = 436.0N * \cos(9°) = 430.6\ N$$

So the D/L ratio can be similarly calculated as 0.1584.
Finding the modified vectors, $\vec{L'}, \vec{D'}, \vec{W'}$

$$\vec{L'} = \begin{bmatrix} 0 \\ 430.6\ N \end{bmatrix} \quad \vec{D'} = \begin{bmatrix} -68.21\ N \\ 0 \end{bmatrix} \quad \vec{W'} = \begin{bmatrix} 436\ N\sin(9°) \\ -436\ N\cos(9°) \end{bmatrix} = \begin{bmatrix} 68.21\ N \\ -430.6\ N \end{bmatrix}$$

This concludes this example. For use with the examples on the attached worksheet, here are a list of useful trigonometric identities.

$$\tan a = \frac{\sin a}{\cos a} \qquad \cot a = \frac{\cos a}{\sin a} \qquad \sec a = \frac{1}{\cos a} \qquad \csc a = \frac{1}{\sin a}$$

$$\sin^2 a + \cos^2 a = 1$$

$$\sin(-a) = -\sin(a) \qquad \cos(-a) = \cos(a) \qquad \tan(-a) = -\tan(a)$$

$$\sin(\alpha+\beta) = \sin\alpha\ \cos\beta + \cos\alpha\ \sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\ \cos\beta - \sin\alpha\ \sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\ \cos\beta - \cos\alpha\ \sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\ \cos\beta + \sin\alpha\ \sin\beta$$

$$\cos a = \sin(90° - a) \qquad \sin a = \cos(90° - a)$$

 Name____________________

Glide Slope II Activity Sheet

1) The 1902 Wright Glider has a mass of 53.06 kg, and during a test "flight" glided at an angle of 7° for a glide length of 152.4 m. Using this information draw the free body diagram, determine the height of the flight, $\vec{L}, \vec{D}, \vec{W}$, and the D/L ratio. (use g = 9.81 m/s^2). For this problem, use the standard (non-rotated) coordinate system to solve.

2) Verify your answers in problem 1 by solving the same problem, using a rotated coordinate system (solve $\vec{L'}, \vec{D'}, \vec{W'}$ too). Which method do you think is easier?

 Name____________________

3) The CG-4 Hadrian glider has a loaded mass of 3402 kg and a total wing area of 83.6 m^2, and during a test "flight" glided at an angle of 4.33°, with an initial height of 7315 m, constant velocity of 26.82 m/s. Assume the air density is 0.7364 kg/m^3, and g=9.81 m/s^2. From this information, determine the glide length, the magnitude and direction of the drag, the direction and magnitude of the lift, and the D/L ratio.

4) Using the information in the previous example, determine c_D and c_L for the CG-4 Hadrian Glider.

5) For a challenge! The rotation that was applied to change the coordinates, **R(θ),** is actually given by a matrix, and maps (or more intuitively, drags) all of the points in the coordinate plane about the origin counterclockwise by some angle **θ.** The matrix that performs this is given below:

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

When we "apply" a transformation to a vector, the following expression is yielded: $\mathbf{R}*\vec{\boldsymbol{x}}$

For exercise 5, use matrix multiplication to prove that rotating the vectors $\vec{L}, \vec{D}, \vec{W}$ via $\mathbf{R}(\boldsymbol{\alpha})*\vec{\boldsymbol{L}}$, $\mathbf{R}(\boldsymbol{\alpha})*\vec{\boldsymbol{D}}$ and $\mathbf{R}(\boldsymbol{\alpha})*\vec{\boldsymbol{W}}$ yields $\vec{L'}, \vec{D'}, \vec{W'}$.

References:
Lift: https://www.grc.nasa.gov/www/k-12/airplane/liftco.html
Drag Coefficient: https://www.grc.nasa.gov/www/k-12/airplane/drageq.html
Rotation Transformation: http://planning.cs.uiuc.edu/node98.html
Trig identities: https://www2.clarku.edu/faculty/djoyce/trig/identities.html
C-47: https://www.nationalmuseum.af.mil/Visit/Museum-Exhibits/Fact-Sheets/Display/Article/196271/douglas-c-47d-skytrain/
WACO Glider: https://www.nationalmuseum.af.mil/Visit/Museum-Exhibits/Fact-Sheets/Display/Article/196272/waco-cg-4a-hadrian/

 Name____KEY___________

Glide Slope II Activity Sheet

1) The 1902 Wright Glider has a mass of 53.06 kg, and during a test "flight" glided at an angle of 7° for a glide length of 152.4 m. Using this information draw the free body diagram, determine the height of the flight, $\vec{L}, \vec{D}, \vec{W}$, and the D/L ratio. (use $g = 9.81$ m/s^2). For this problem, use the standard (non-rotated) coordinate system to solve.

$$h = 18.57\ m$$

$$\vec{L} = \begin{bmatrix} 62.96\ N \\ 512.75\ N \end{bmatrix}, L = 516.6\ N$$

$$\vec{D} = \begin{bmatrix} -62.96\ N \\ 7.73\ N \end{bmatrix}, D = 63.44\ N$$

$$\vec{W} = \begin{bmatrix} 0\ N \\ -520.5\ N \end{bmatrix}, W = 520.5\ N$$

$$\frac{D}{L} = 0.1228$$

2) Verify your answers in problem 1 by solving the same problem, using a rotated coordinate system (solve $\vec{L'}, \vec{D'}, \vec{W'}$ too). Which method do you think is easier?

$$h = 18.57\ m$$

$$\vec{L'} = \begin{bmatrix} 0\ N \\ 516.6\ N \end{bmatrix}, L = 516.6\ N$$

$$\vec{D'} = \begin{bmatrix} -63.43\ N \\ 0\ N \end{bmatrix}, D = 63.43\ N$$

$$\vec{W'} = \begin{bmatrix} 63.43\ N \\ -516.6\ N \end{bmatrix}, W = 520.5\ N$$

$$\frac{D}{L} = 0.1228$$

Often times, students will prefer this method, as there are less steps, but the intuition of rotation may be difficult. Emphasis should be made to indicate, also, the modified vectors should be designated on their figures or states as "7 degrees clockwise from the axis"

Name______KEY________

3) The CG-4 Hadrian glider has a loaded mass of 3402 kg and a total wing area of 83.6 m^2, and during a test "flight" glided at an angle of 4.33° with an initial height of 7315 m and constant velocity of 26.82 m/s. Assume the air density is 0.7364 kg/m^3, and g=9.81 m/s^2. From this information, determine the glide length, the magnitude and direction of the drag, the direction and magnitude of the lift, and the D/L ratio.
Glide length = 96 890m. L = 33.28 kN directed at 86.67° (see below for vector notation) D = 2.520 kN directed at 176.67° D/L ratio is 0.07572.

$$\vec{L} = \begin{bmatrix} 2.512\ kN \\ 33.18\ kN \end{bmatrix} \quad \vec{D} = \begin{bmatrix} -2.512\ kN \\ 0.190\ kN \end{bmatrix}$$

4) Using the information in the previous example, determine c_D and c_L for the CG-4 Hadrian Glider.

$C_D = 0.1138$, $C_L = 1.503$

5) For a challenge! The rotation that was applied to change the coordinates, **R(θ),** is actually given by a matrix, and maps (or more intuitively, drags) all of the points in the coordinate plane about the origin counterclockwise by some angle **θ.** The matrix that performs this is given below:

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

When we "apply" a transformation to a vector, the following expression is yielded: **R***$\boldsymbol{\vec{x}}$
For exercise 5, use matrix multiplication to prove that rotating the vectors $\vec{L}, \vec{D}, \vec{W}$ via **R(α)***$\boldsymbol{\vec{L}}$**,** **R(α)***$\boldsymbol{\vec{D}}$ and **R(α)***$\boldsymbol{\vec{W}}$ yields $\vec{L'}, \vec{D'}, \vec{W'}$.

$$\boldsymbol{R}(\boldsymbol{\alpha}°) * \boldsymbol{\vec{L}} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} * \begin{bmatrix} L\cos(90-\alpha) \\ L\sin(90-\alpha) \end{bmatrix} = \begin{bmatrix} L\cos(\alpha)\cos(90-\alpha) - L\sin(\alpha)\sin(90-\alpha) \\ L\sin(\alpha)\cos(90-\alpha) + L\cos(\alpha)\sin(90-\alpha) \end{bmatrix}$$

$$using\ cos(A+B) = cos(A)cos(B) - sin(A)sin(B)\ ;\ and\ sin(A+B) = sin(A)cos(B) + cos(A)sin(B)$$

$$\boldsymbol{R}(\boldsymbol{\alpha}°) * \boldsymbol{\vec{L}} = \begin{bmatrix} L(cos(\alpha+90-\alpha)) \\ L(\sin(\alpha+90-\alpha)) \end{bmatrix} = \begin{bmatrix} L\cos(90°) \\ L\sin(90°) \end{bmatrix} = \begin{bmatrix} 0 \\ L \end{bmatrix}$$

The other solutions can be gathered using similar identities.

Mathematics of flight:

Glide Slope II

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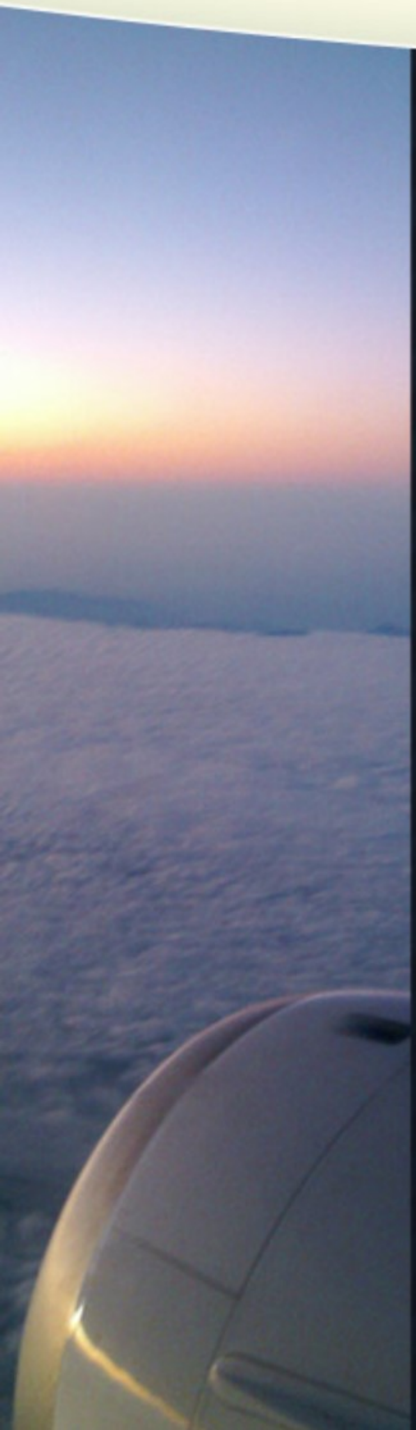


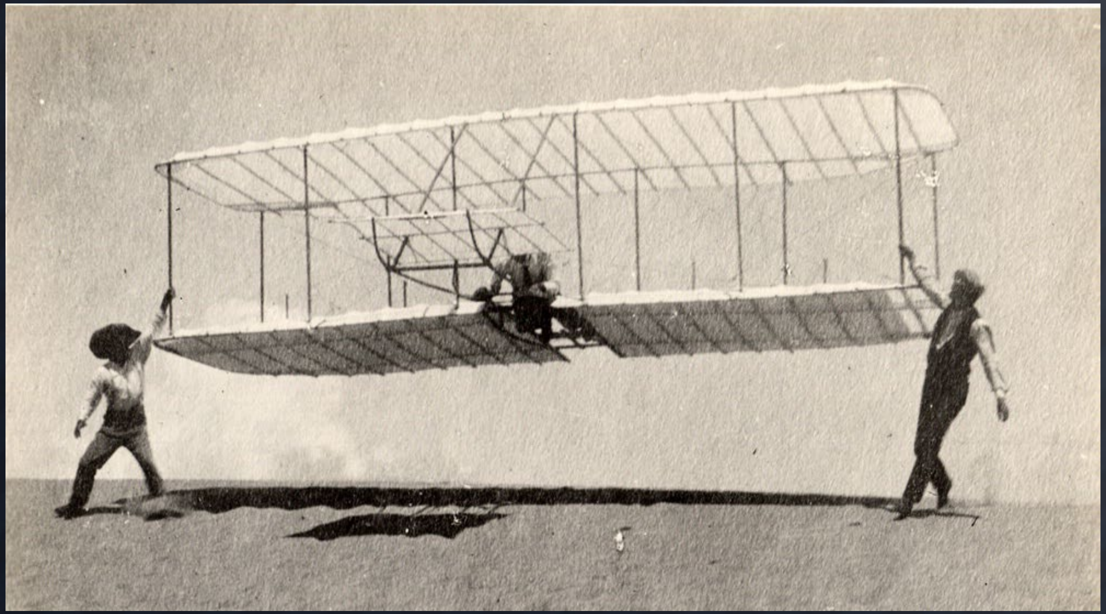
Glide Slope II

The students will:

- Be introduced to formulas used in flight related to navigation and aircraft performance.
- Learn to calculate the glide slope using trigonometry.
- Vector components and forces in vector form
- Learn about the proportionality of the drag and lift coefficients, and how they are then related to the glide slope.

Glide Slope II

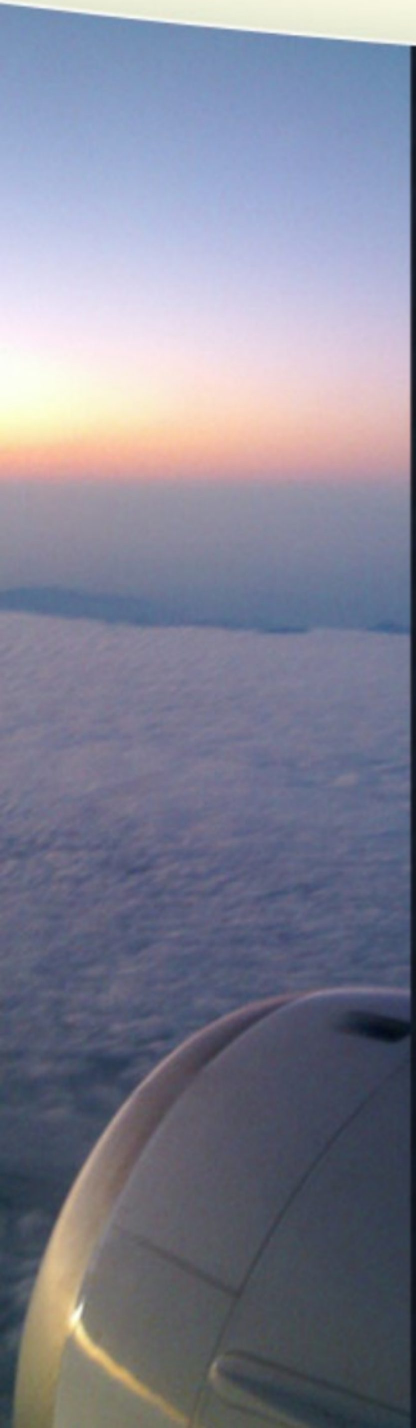
A glider is a special kind of aircraft that has no engine. The Wright brothers perfected the design of the first airplane and gained piloting experience through a series of glider flights from 1900 to 1903.



With Wilbur Wright at the controls, Dan Tate, left, and Edward C. Huffaker launch the 1901 Wright Glider at Kitty Hawk, N.C.
Credit: Library of Congress, courtesy National Air and Space Museum, Smithsonian Institution

Glide Slope II

During World War II, gliders such as the WACO CG-4 were towed aloft by C-47 and C-46 aircraft then cut free to glide over many miles.



Glide Slope II

If a glider is in a steady (constant velocity, no acceleration) descent, the forces acting on the plane can be considered equal. The flight path intersects the ground at an angle (α) called the ***glide angle***. If we know the distance flown and the altitude change, the glide angle can be calculated using trigonometry.

The tangent (**tan**) of the glide angle (α) is equal to the change in height (***h***) divided by the distance flown (***d***):

$$\tan(\alpha) = h / d$$

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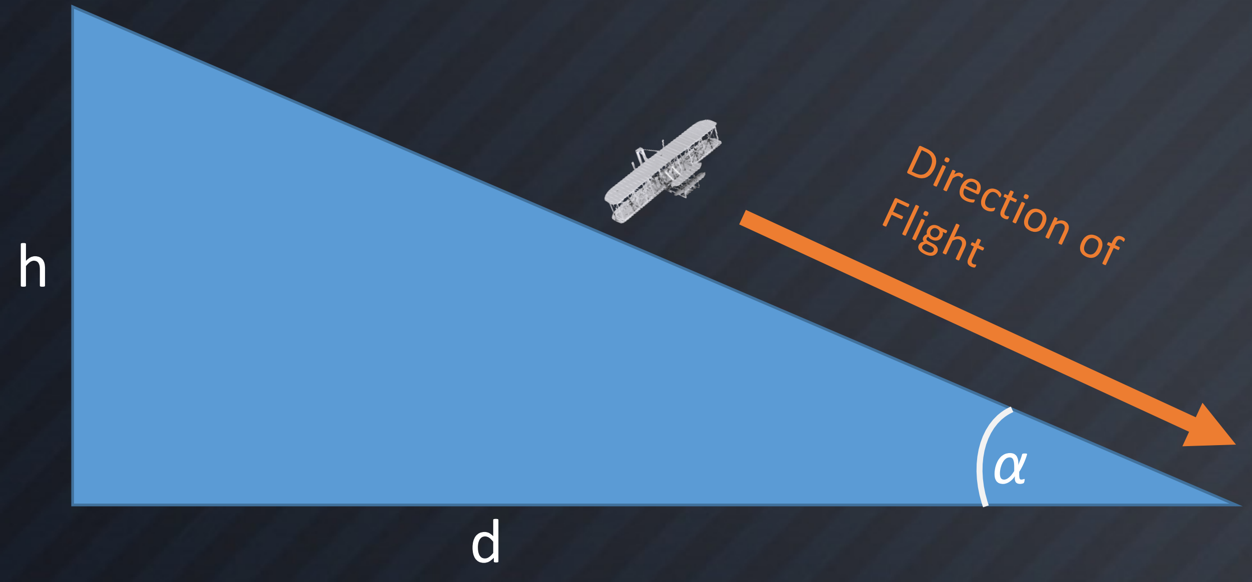
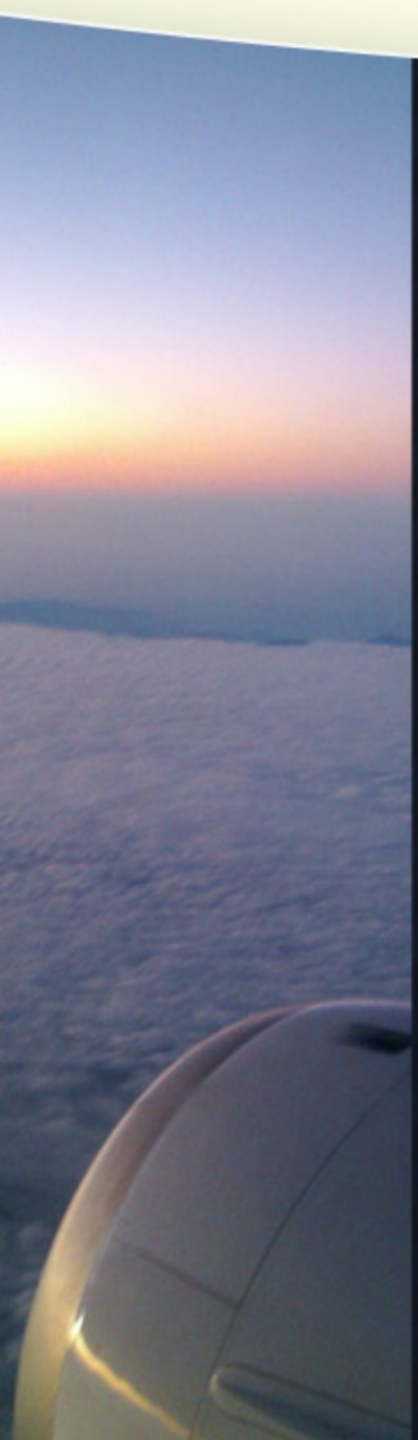
Glide Slope II



d = horizontal distance flown
h = change in height
α = glide angle

From trigonometry: $\tan(\alpha) = h/d$



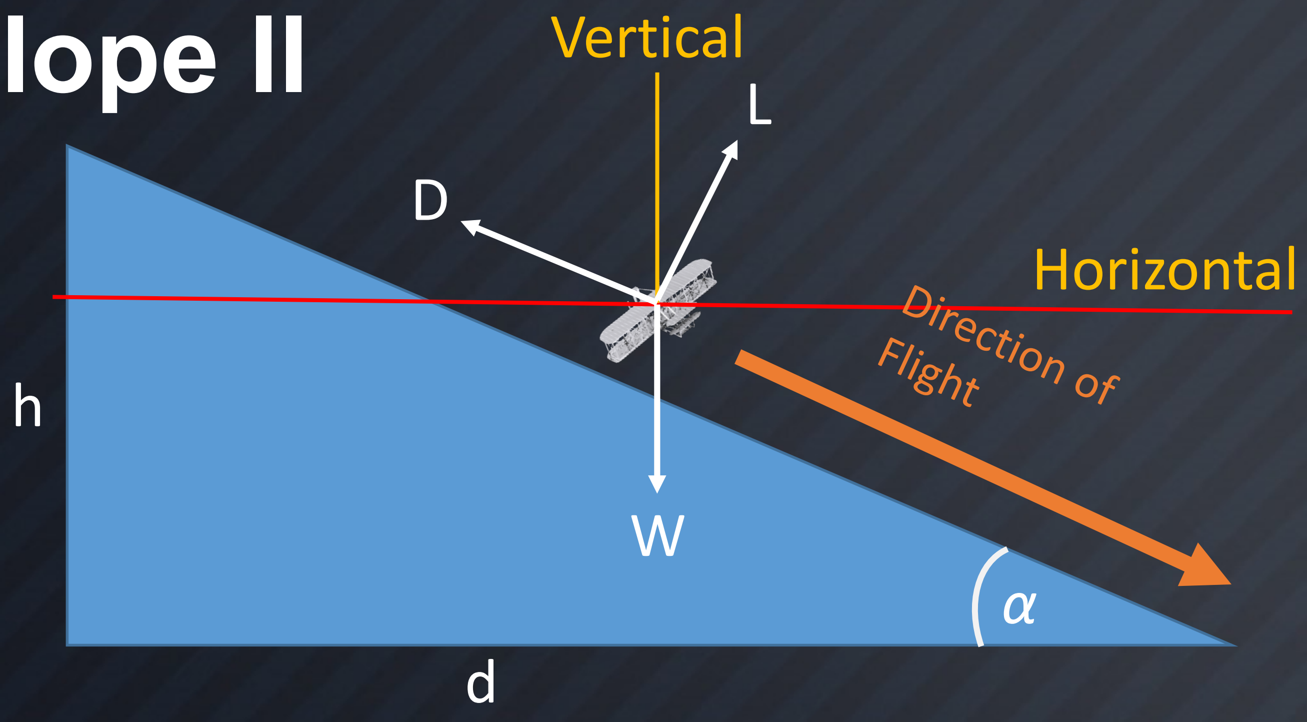
Glide Slope II

There are three forces acting on the glider; lift, weight, and drag. The weight of the glider is given by the symbol "***W***" and is directed vertical, toward the center of the earth. The weight is then perpendicular to the horizontal red line drawn parallel to the ground and through the center of gravity.

The drag of the glider is designated by "***D***" and acts along the flight path opposing the motion.

Lift, designated "***L***" acts perpendicular to the flight path. Using some geometry theorems on angles, perpendicular lines, and parallel lines, we see the glide angle " α " also defines the angle between the lift and the vertical, and between the drag and the horizontal.

Glide Slope II



L = Lift
D = Drag
W = Weight
α = glide angle

Vertical Equation: $L\cos(\alpha) + D\sin(\alpha) = W$

Horizontal Equation: $L\sin(\alpha) = D\cos(\alpha)$

$$\frac{\sin(\alpha)}{\cos(\alpha)} = \tan(\alpha) = \frac{D}{L}$$

*Assuming velocity is constant

Glide Slope II

Assuming that the forces are balanced (no acceleration of the glider), we can write two vector component equations for the forces.

$$\vec{L} = \begin{bmatrix} L\cos(90-\alpha) \\ L\sin(90-\alpha) \end{bmatrix} \quad \vec{D} = \begin{bmatrix} D\cos(180-\alpha) \\ D\sin(180-\alpha) \end{bmatrix} \quad \vec{W} = \begin{bmatrix} W\cos(270) \\ W\sin(270) \end{bmatrix} = \begin{bmatrix} 0 \\ -W \end{bmatrix}$$

Since the forces assumed to be balanced, the sum of forces must be equal to zero in both the x and the y directions. That is:

$$\vec{L} + \vec{D} + \vec{W} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So we end up with a system of equations, after using some trig. identities

$$L\cos(\alpha) + D\sin(\alpha) = W$$

$$L\sin(\alpha) = D\cos(\alpha)$$

The second equation can also be written as the following

$$\frac{D}{L} = \tan(\alpha)$$

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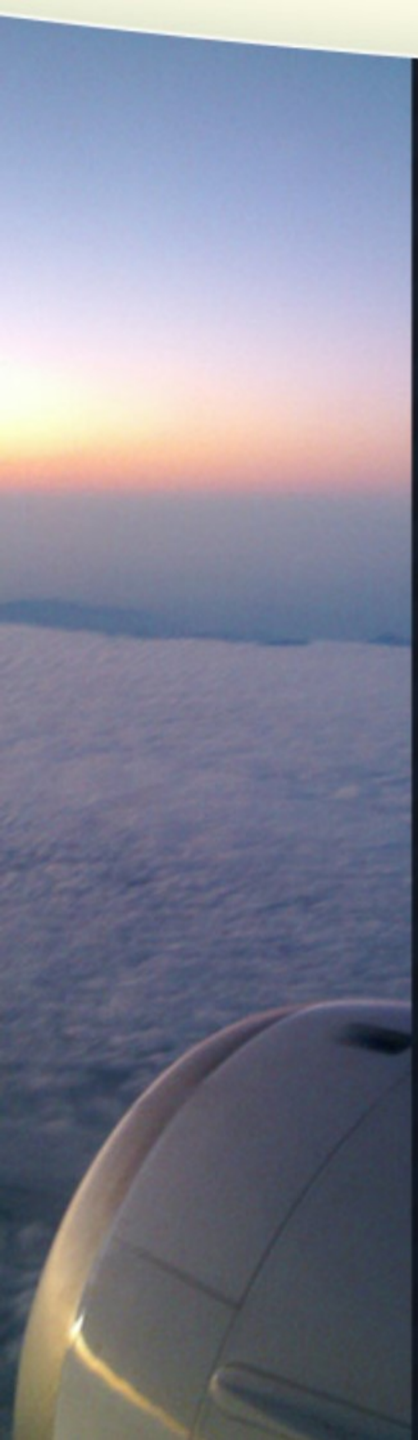


Glide Slope II

The lift equation:

$$Lift = \frac{1}{2} C_l * \rho * v^2 * A$$

Lift coefficient (c_L) is a unitless coefficient that factors in many of the intricacies in flight. In practice, it is experimentally determined.

In this equation, the lifting force is given in Newtons (N), *ρ* is the air density (in kg/m^3), *v* is the velocity of the aircraft (in m/s), and *A* is the area of the lifting body (e.g. wing, in m^2).

Glide Slope II

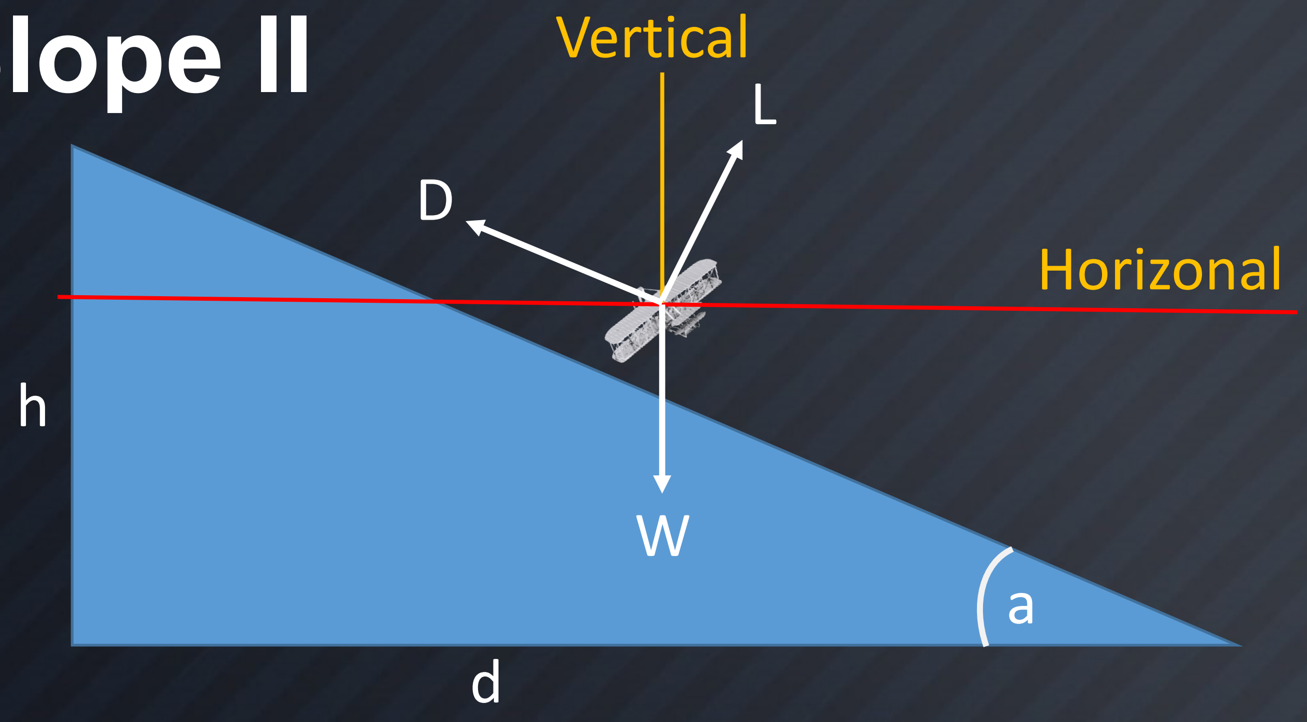
The drag equation:

$$Drag = \frac{1}{2} C_D * \rho * v^2 * A$$

Drag coefficient (c_D) is a unitless coefficient that factors in many of the intricacies in flight. In practice, it is experimentally determined.

In this equation, the drag force is given in Newtons (N), ρ is the air density (in kg/m^3), v is the velocity of the aircraft (in m/s), and A is the area of the lifting body (e.g. wing, in m^2).

Glide Slope II



cl = Lift coefficient
cd = Drag coefficient

$$\frac{Drag}{Lift} = \frac{\cancel{\frac{1}{2}} c_D \cancel{* \rho * v^2 * A}}{\cancel{\frac{1}{2}} c_L \cancel{* \rho * v^2 * A}} = \frac{c_D}{c_L} = \tan(\alpha)$$

Glide Slope II

If we use algebra to re-arrange the horizontal force equation we find that the drag divided by the lift is equal to the sine of the glide angle divided by the cosine of the glide angle. This ratio of trigonometric functions is equal to the tangent of the angle.

$$D / L = \sin(\alpha) / \cos(\alpha) = \tan(\alpha)$$

We can use the drag equation and the lift equation to relate the glide angle to the drag coefficient (**c_D**) and lift coefficient (**c_L**) that the Wrights measured in their wind tunnel tests.

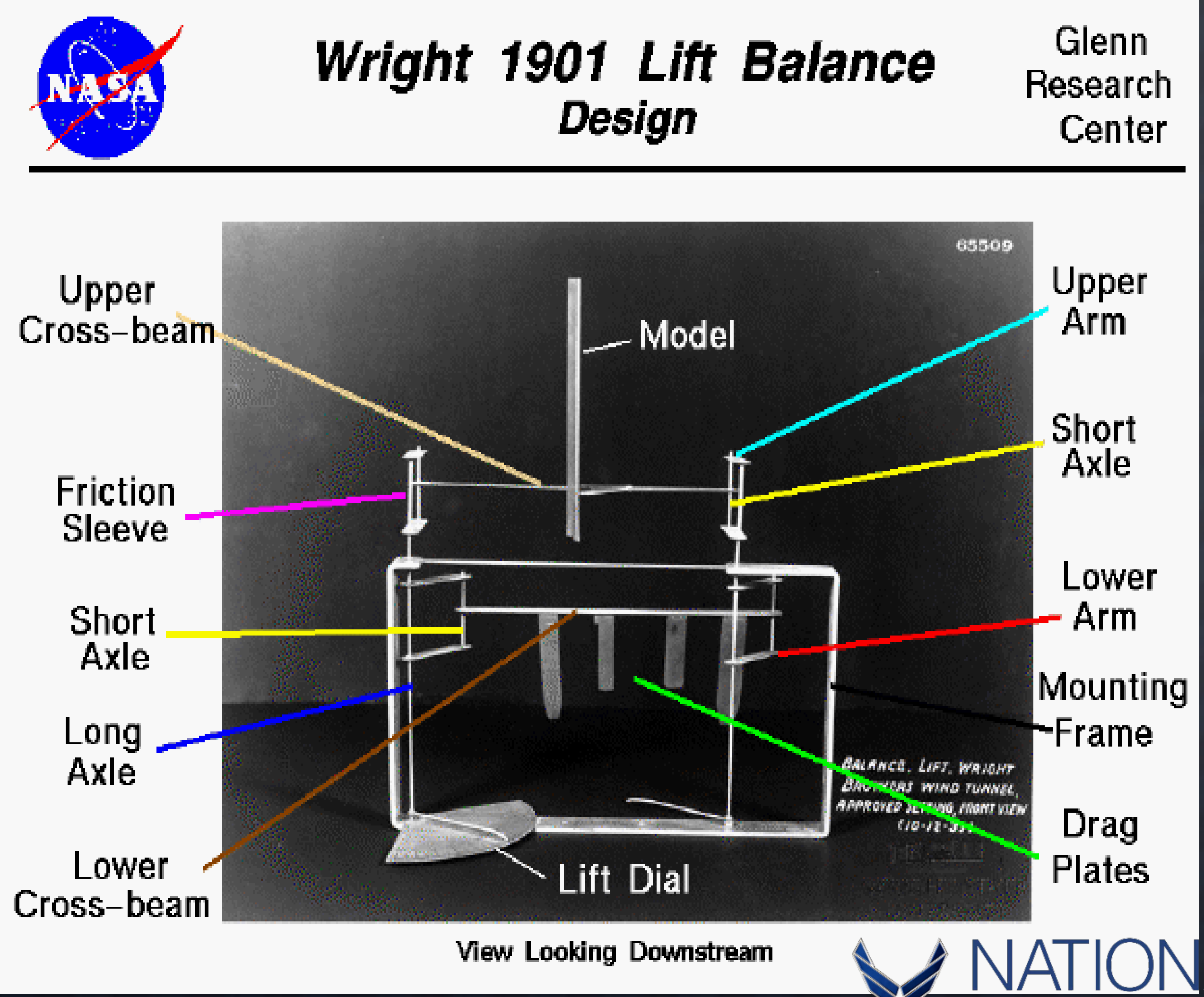
$$D / L = c_D / c_L = \tan(\alpha)$$

Glide Slope II

This is a replica of the wind tunnel designed and built by the Wright Brothers in the fall of 1901 to test airfoil designs. The blower fan, driven by an overhead belt, produced a 25 to 35 mph wind for testing the lift of various planes and curved surfaces. Aerodynamic tables derived from these tests were vital to the successful design of the Wright 1903 Kitty Hawk airplane. Inside the tunnel is a model of a Wright lift balance used to measure the lift of a test surface. The wind tunnel replica was constructed under the personal supervision of Orville Wright prior to World War II.

Wright Brothers
1901 Wind Tunnel

Glide Slope II



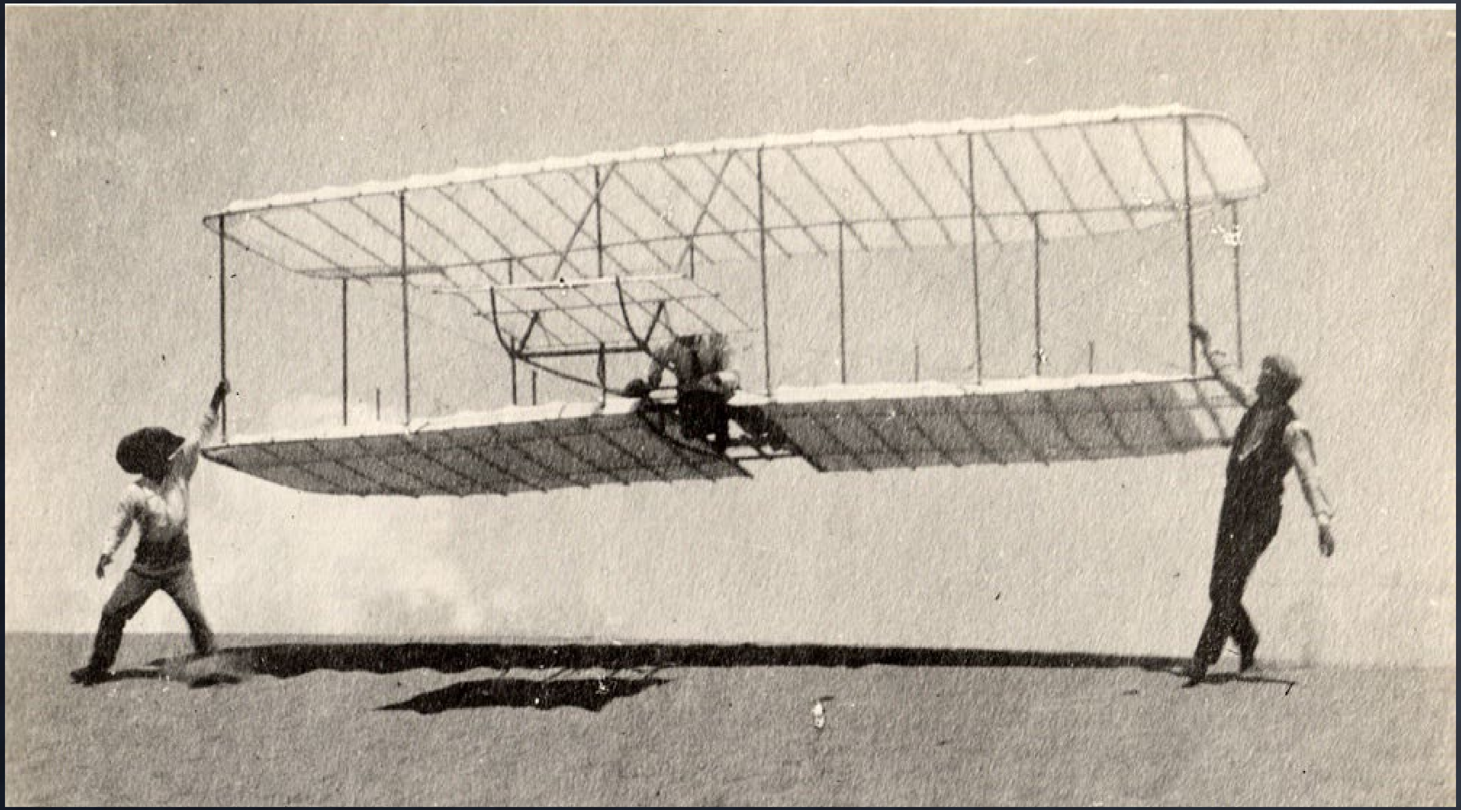
Glide Slope II

During the operation of the drag balance the brothers made measurements of the effects of wing design on glide angle through the drag to lift ratio.

The inverse of the drag to lift ratio is the L/D ratio which is an efficiency factor for aircraft design.

The higher the L/D, the lower the glide angle, and the greater the distance that a glider can travel across the ground for a given change in height.

Glide Slope II



With Wilbur Wright at the controls, Dan Tate, left, and Edward C. Huffaker launch the 1901 Wright Glider at Kitty Hawk, N.C.
Credit: Library of Congress, courtesy National Air and Space Museum, Smithsonian Institution

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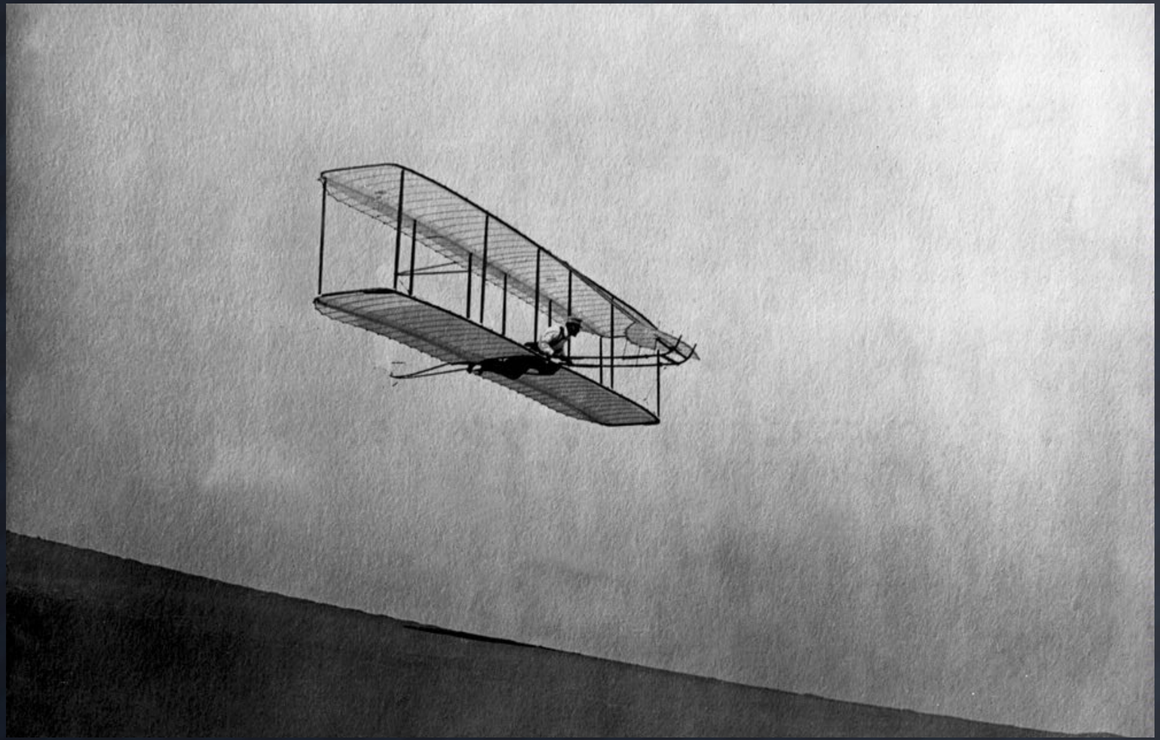
Glide Slope II



Wilbur Wright gliding in 1902. The Wrights added a vertical tail to their glider to deal with the lateral control problems experienced in 1901. The more graceful appearance of the 1902 machine over the previous gliders is evident in this picture.
Credit: National Air and Space Museum, Smithsonian Institution

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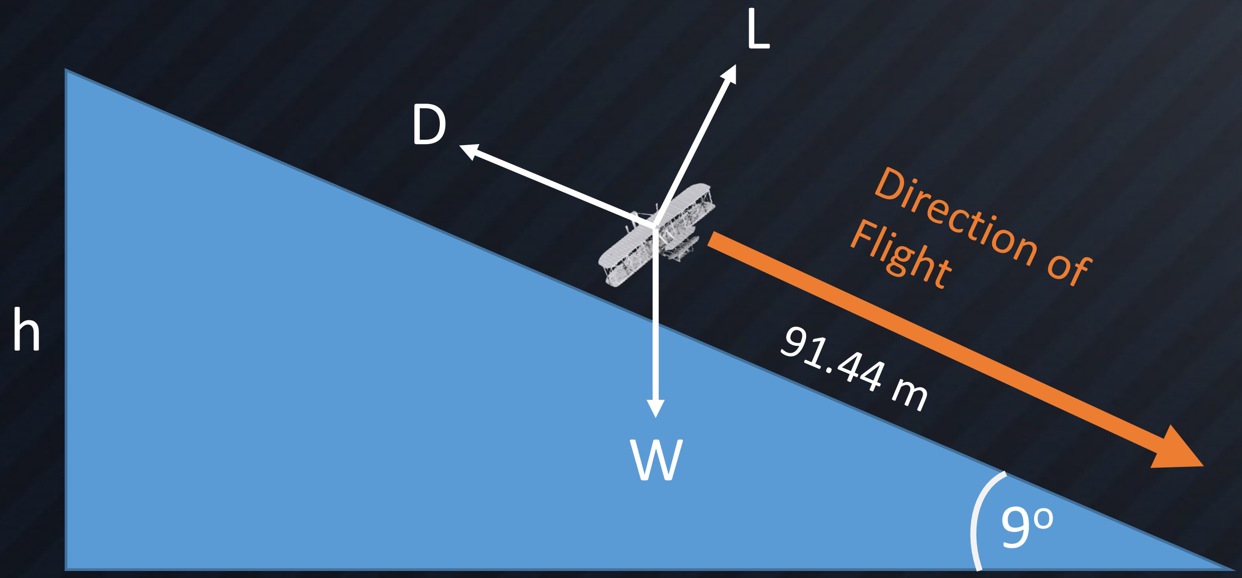
Glide Slope II

Example: The 1901 Wright Glider has a mass of 44.44 kg, and during a test "flight" glided at an angle of 9° for a glide length of 91.44 m. Using this information draw the free body diagram, determine the height of the flight, $\vec{L}, \vec{D}, \vec{W}$, and the D/L ratio. (use g = 9.81 m/s^2).

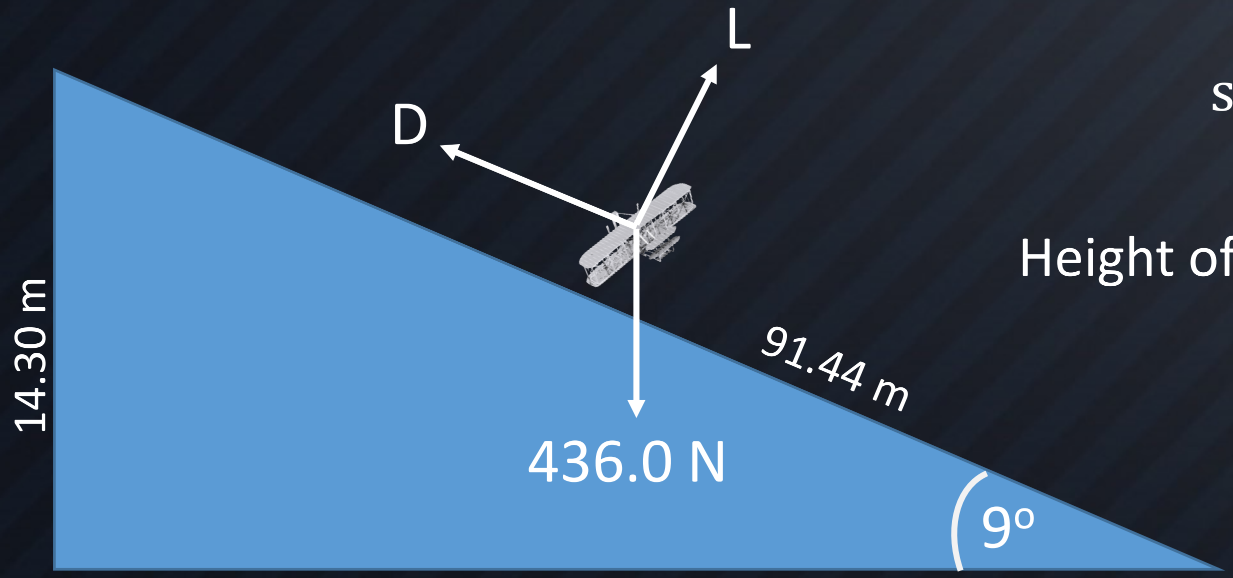
Lastly, confirm that $\vec{L} + \vec{D} + \vec{W} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (the sum of forces is zero)



Glide Slope II

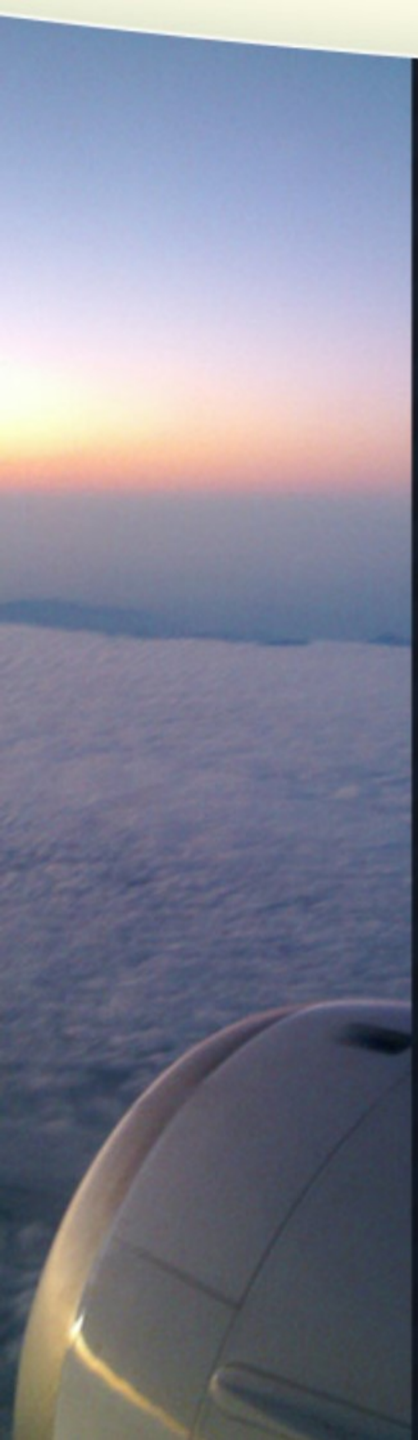
Example: The 1901 Wright Glider has a mass of 44.44 kg, and during a test "flight" glided at an angle of 9° for a glide length of 91.44 m. Using this information draw the free body diagram, determine the height of the flight, $\vec{L}, \vec{D}, \vec{W}$, and the D/L ratio. (use g = 9.81 m/s^2).

W = 44.44 kg * 9.81 m/s^2 = 436.0 N



$$\sin(9°) = \frac{h}{91.44\ m}$$

Height of the glider = 14.30 m



Glide Slope II

$$L\cos(9°) + D\sin(9°) = 436.0N$$

$$\frac{D}{L} = \tan(\alpha) \rightarrow D = L\tan(9°)$$

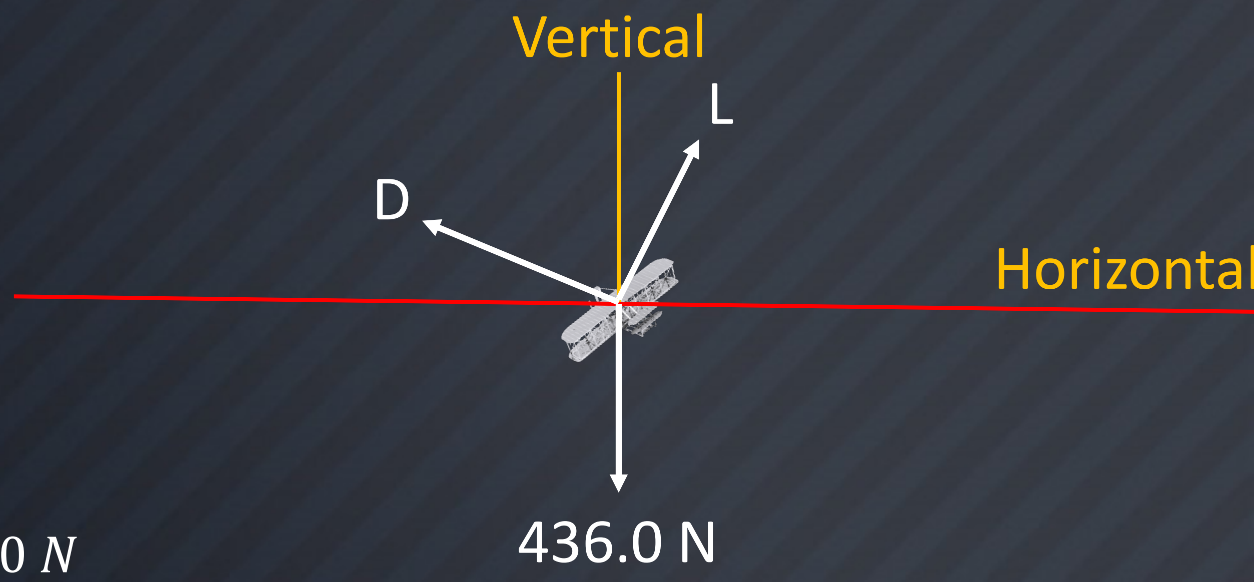
Thus by substitution

$$L\cos(9°) + L\tan(9°)\sin(9°) = 436.0\ N$$

Rearranging and solving yields

$$L = \frac{436.0N}{\cos(9°) + \tan(9°)\sin(9°)} = 430.6\ N$$

$$D = 430.6\ N * \tan(9°) = 68.21\ N\ so\ {}^{D}/_{L} = 0.1584$$



$$L\cos(\alpha) + D\sin(\alpha) = W$$

$$L\sin(\alpha) = D\cos(\alpha)$$

$$\frac{D}{L} = \tan(\alpha)$$

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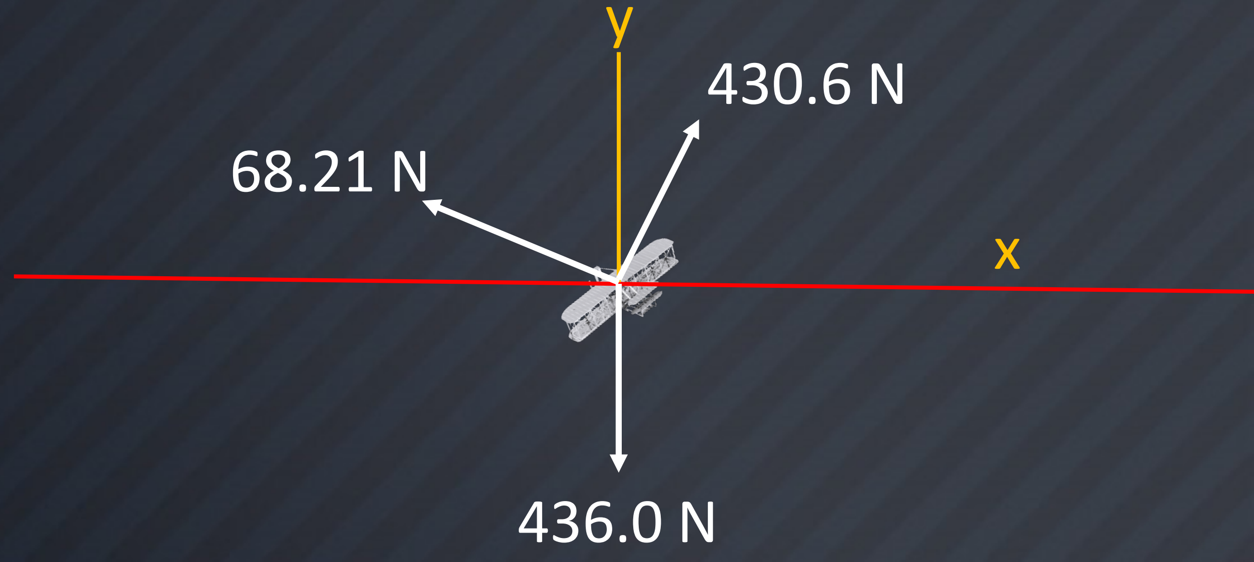
Glide Slope II



Given α = 9º

$$\vec{L} = \begin{bmatrix} L\ \cos(90-9) \\ L \sin(90-9) \end{bmatrix}$$

$$\vec{L} = \begin{bmatrix} 430.6N \cos(81°) \\ 430.6N \sin(81°) \end{bmatrix} = \begin{bmatrix} 67.36N \\ 425.3N \end{bmatrix}$$

$$\vec{D} = \begin{bmatrix} D \cos(180-9) \\ D \sin(180-9) \end{bmatrix}$$

$$\vec{D} = \begin{bmatrix} 68.21N \cos(171°) \\ 68.21N \sin(171°) \end{bmatrix} = \begin{bmatrix} -67.36N \\ 10.7N \end{bmatrix}$$

$$\vec{W} = \begin{bmatrix} 0 \\ -436\ N \end{bmatrix}$$

$$\vec{L} = \begin{bmatrix} L\ \cos(90-\alpha) \\ L \sin(90-\alpha) \end{bmatrix}$$

$$\vec{D} = \begin{bmatrix} D \cos(180-\alpha) \\ D \sin(180-\alpha) \end{bmatrix}$$

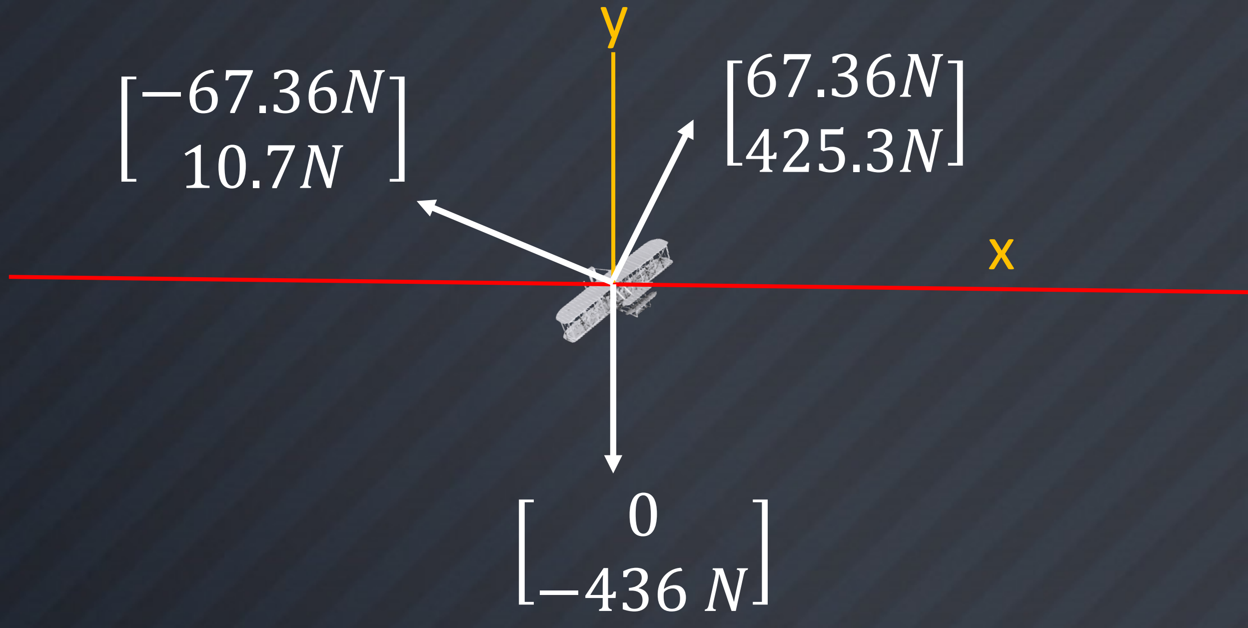
$$\vec{W} = \begin{bmatrix} W \cos(270) \\ W \sin(270) \end{bmatrix} = \begin{bmatrix} 0 \\ -W \end{bmatrix}$$

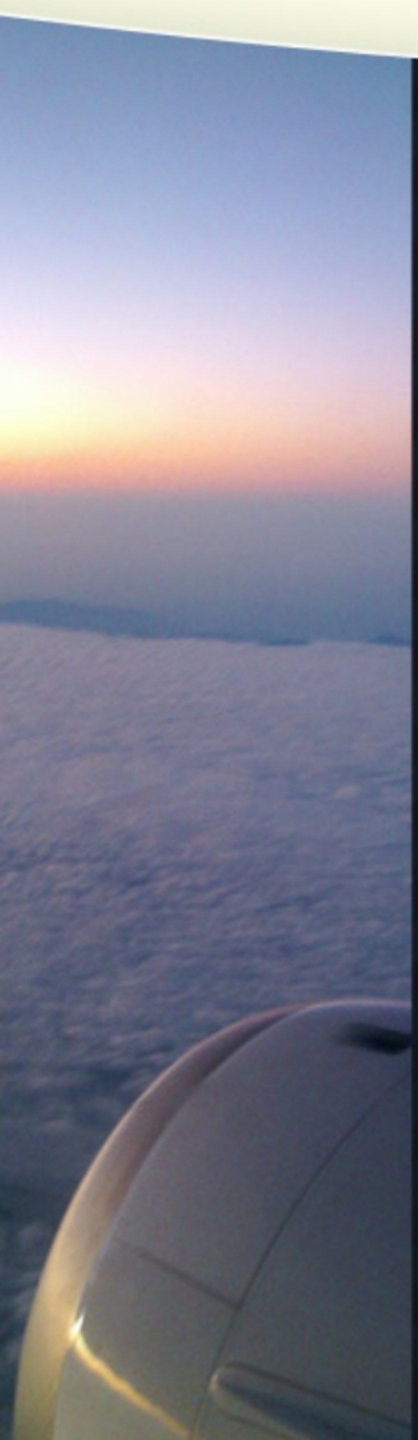
Glide Slope II

Finally check $\vec{L} + \vec{D} + \vec{W}$

$$\begin{bmatrix} 67.36N \\ 425.3N \end{bmatrix} + \begin{bmatrix} -67.36N \\ 10.7N \end{bmatrix} + \begin{bmatrix} 0 \\ -436\ N \end{bmatrix}$$

$$\begin{bmatrix} 67.36\text{N} \ - 67.36\text{N} \\ 425.3\text{N} + 10.7\text{N} - 436\text{N} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$





Glide Slope II

Today we discussed:

- The formulas used in flight related to navigation and aircraft performance.
- Learned how to calculate the glide slope using trigonometry.
- Vector components and forces in vector form
- Learn about the proportionality of the drag and lift coefficients, and how they are then related to the glide slope.