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Mathematics of Flight: Headwinds and Tailwinds

Students will practice use of systems of equations with two unknowns (variables) by learning about the effects of tailwinds and headwinds on aircraft flight.

Grade Level: 7-8

Ohio Learning Standards/Science (2018)

Physical Science

PS.FM.1 Motion

Ohio Learning Standards/Mathematics (2017)

Equations and Expressions

7.EE.1 Apply properties of operations on linear functions
7.EE.2 Rewriting expressions in equivalent form
7.EE.3 Solve multistep real-life problems.
7.EE.4 Use variables to represent quantities in the real world
8.EE.7 Solve linear equations in one variable

Materials Required:

- Paper
- Writing utensil
- Calculator

LESSON PLAN

Lesson Objectives

The students will:

- Be introduced to formulas used in flight, related to navigation and aircraft performance.
- Learn to calculate effects of headwinds and tailwinds on speed.
- Briefly learn about vectors in one dimension.

Goal

In this lesson, students will gain an understanding of common calculations performed by flight personnel. Students should also develop an understanding of solving linear equations of two unknowns, modelling problems, and one dimensional vectors.

Background

Crosswinds are any winds that have an effect on an aircraft's flight path. There are special cases such as **headwinds**, where the wind acts opposite to the planes direction. Other special cases include a **tailwind**, where the plane and wind are acting in the same direction. As you may suspect, the speed of the aircraft *increases when there is a tailwind* and *decreases when there is a headwind*. This effect is not limited to aircraft – outdoor sports such as soccer, football, and many others are impacted by this effect as well.

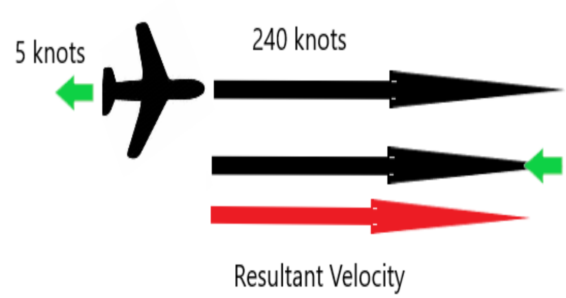
Example:

With a headwind, an aircraft travels 235 nautical miles in an hour. With a tailwind, the aircraft travels 245 nautical miles in an hour. Determine the speed of the wind, and speed of the aircraft in still air. Knots are units of speed which are nautical miles per hour.

Solution:

When you have constant speed, the formula for uniform motion is

$$distance(d) = rate * time = r * t$$



In a headwind, we know that the wind speed is decreasing our initial speed (we'll call it *s*) by some amount (we'll call it *w*). So the rate in the headwind can be given by:

$$rate\ (headwind) = \ s - w$$

In a tailwind, we know that the wind speed is increasing our initial speed (same *s* as before) by some about (the same *w* as before).

$$rate\ (tailwind) = s + w$$

Now, given the information from the sample problem, we can solve since we know the distance and the time. We can come up with a number for the rate in both cases.

$$rate = \frac{distance}{time}$$

$$rate\ (headwind) = \frac{235\ nautical\ miles}{1\ hour} = 235\ knots = s - w$$

$$rate(tailwind) = \frac{245\ nautical\ miles}{1\ hour} = 245\ knots = s + w$$

There are two ways that we can approach this problem. One of them is by adding the two equations that we have together to try to eliminate one of the unknowns. If we add the top equation to the bottom equation, we get:

$$235\ knots + 245\ knots = s - w + s + w$$
$$480\ knots = 2s \rightarrow 240\ knots = s$$

Then we are able to substitute our value for *s* into either of the two rate equations:

$$235\ knots = 240\ knots - w\ or\ 245\ knots = 240\ knots + w$$

And we determine the wind speed to be **5 knots**, and the speed of the aircraft is **240 knots**.

The other way to approach this problem would be to solve for one variable, and then substitute that into the other equation we started with, and solve accordingly. We will employ this method in the next example.

Example 2
With a headwind, an aircraft travels 300 nautical miles in 45 minutes. With a tailwind, the aircraft travels 230 nautical miles in 30 minutes. Find the speed of the aircraft in still wind.

Solution
Now, we have two different time intervals and they are given in minutes instead of hours. We will first convert minutes to hours, and then determine speed in the headwind and tailwind.

$$45\ minutes * \frac{1\ hour}{60\ minutes} = \frac{3}{4}\ hour, and\ 30\ minutes * \frac{1\ hour}{60\ minutes} = \frac{1}{2}\ hour$$

Thus,

$$rate(tailwind) = \frac{230\ nautical\ miles}{1/2\ \ hours} = 460\ knots =\ s + w$$

$$rate(headwind) = \frac{300\ nautical\ miles}{3/4\ hours} = 400\ knots = s - w$$

Now, instead of adding the two equations, let us take the equation for the rate of the headwind, and solve for the variable *s*.

$$400\ knots =\ s - w \xrightarrow{add\ w\ on\ both\ sides} 400\ knots + w = s$$

Now, we take this equation and plug it into the rate (tailwind) equation:

$$rate(tailwind) = 460 = s + w \xrightarrow{plug\ in\ s=400+w} 460 = 400 + w + w = 460 - 400 = 2w$$

$$60\ knots = 2w \rightarrow w = 30\ knots$$

Then we can use the equation we developed for *s*.

$$s = 400\ knots + w = 400\ knots + 30\ knots$$

$$\boldsymbol{s = 430\ knots}$$

Lastly, it is important to note that **speed** is a number, while **velocity** is a **vector**. Vectors have a *magnitude* and a *direction*. If we know that an aircraft was going 430 knots forward, then we would know not only its speed, but also its velocity. A tailwind is a vector, since we know it is going in the same direction as the aircraft; likewise, a headwind is a vector, since we know it is opposing the direction of the aircraft.

For Additional References and Activities visit our website!
https://www.nationalmuseum.af.mil/Education/

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Round answers to the nearest hundredth. Sketch figures as necessary.

1) With a tailwind, an aircraft travels 190 nautical miles in 30 minutes. With a headwind, it travels 155 nautical miles in 60 minutes. Find the speed of the aircraft in still air and the speed of the wind.

2) With a headwind, an aircraft travels 270 nautical miles in 45 minutes. With a tailwind, the aircraft travels 230 nautical miles in 30 minutes. Find the speed of the aircraft in still air.

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3) Two pilots are flying cross-country, but starting on opposite sides. One of the pilots experiences a headwind of 15 knots the entire way, while the other pilot experiences a tailwind of 15 knots for the entire distance. In still air, both of their planes have a max speed of 500 knots. If the flight length is 3000 nautical miles, how long will each of their flights take, and how much sooner will the one arrive before the other?

4) Two pilots are flying cross-country, but starting on opposite sides. Pilot A experiences a headwind of 15 knots the first 2 hours of flight. After the first two hours, Pilot A experiences a tailwind of 22 knots for the remainder of the flight. The opposite is true for Pilot B, who experiences a tailwind of 15 knots for the first 2 hours, and then has a headwind for the duration of the flight. In still air, both of their planes have a max speed of 500 knots. What must the flight distance be for the two aircraft to land at the same time?

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Mathematics of Flight: Headwinds and Tailwinds

Round answers to the nearest hundredth. Sketch figures as necessary.

1) With a tailwind, an aircraft travels 190 nautical miles in 30 minutes. With a headwind, it travels 155 nautical miles in 60 minutes. Find the speed of the aircraft in still air and the speed of the wind.

The plane's speed in still air is 267.5 knots, and the wind speed is 112.5 knots

2) With a headwind, an aircraft travels 270 nautical miles in 45 minutes. With a tailwind, the aircraft travels 230 nautical miles in 30 minutes. Find the speed of the aircraft in still air.

The plane's speed in still air is 433.33 knots, and the wind speed is 96.67 knots

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3) Two pilots are flying cross-country, but starting on opposite sides. One of the pilots experiences a headwind of 15 knots the entire way, while the other pilot experiences a tailwind of 15 knots for the entire distance. In still air, both of their planes have a max speed of 500 knots. If the flight length is 3000 nautical miles, how long will each of their flights take, and how much sooner will the one arrive before the other (in hours and minutes)?

With a tailwind, it will take 5:47. With a headwind, 6:11. The pilot with a tailwind arrives first by 24 minutes.

4) Two pilots are flying cross-country, but starting on opposite sides. Pilot A experiences a headwind of 15 knots the first 2 hours of flight. After the first two hours, Pilot A experiences a tailwind of 22 knots for the remainder of the flight. The opposite is true for Pilot B, who experiences a tailwind of 15 knots for the first 2 hours, and then has a headwind for the duration of the flight. In still air, both of their planes have a max speed of 500 knots. What must the flight distance be, for the two aircraft to land at the same time?

The time for the trip is 2+1.36 hr = 3:22. The total trip length is 1681.82 nautical miles.