NATIONAL MUSEUM OF THE UNITED STATES AIR FORCE™

# Mathematics with the Altitude of an Aircraft

*Students will practice math skills (functions, algebra, geometry) while learning about one way to determine the altitude of an aircraft in flight.*

LESSON PLAN

**Learning Objectives:**

The students will:

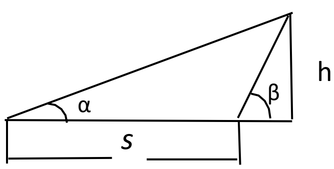
- Be introduced to formulas used in flight related to navigation and aircraft performance.
- Learn to calculate the altitude of an aircraft in flight.

**Purpose:**

Students will gain an understanding of common calculations performed by flight personnel. Students will learn how to work with trigonometric equations and solve real world problems to calculate the altitude of an aircraft in flight.

**Introduction**:

A formula used to determine the altitude on an aircraft in flight being observed by two tracking stations, when given the distance between tracking stations (*s*) and the angles of inclination (α and β) is:




$$h = \frac{s}{\cot a - \cot B}$$

With this equation, we are able to track the varying altitudes of flight for different aircraft. For more information and instruction on this topic, see the attached PowerPoint.

**Grade Level: 9 - 12**

**Ohio Learning Standards/Mathematics (2017)**

*Algebra*
A.SSE.1: Interpret expressions in terms of its context
A.SSE.2: Use the structure of an expression to rewrite it

*Functions*
(+) F.TF.3: Use special triangles to find trigonometric values
(+) F.TF.7: Use inverse functions to solve trigonometric functions
(+) F.TF.9: Prove the addition and subtraction of trigonometric functions

*Geometry*
G.SRT.7: Explain and use the relationships between trigonometric functions

**Materials Required:**

- Paper
- Pencil or Pen
- Calculator
- Computer/Internet for calculations
- Trigonometry Tables

**Procedure:**

***A. Warm-Up***

1. Review the purpose, introduction and attached PowerPoint discussing the meaning and importance behind these equations.

***B. Activity***

1. Students should receive three items for this activity: packet of equations and examples, pencil, and calculator.
2. Students should be able to work with the different equations and solve them effectively.
3. Give students a set amount of time to complete each exercise.
4. NOTE: Questions 3 & 4 may require internet access to solve.
5. Review answers after each question and discuss the best methods to solve the equations.

**Assessment/Evaluation:**

Students will be evaluated on their ability to solve equations and undefined variables. They will be questioned on their problem solving processes and their critical thinking.

**Resources/References**:

Trigonometry functions:

https://www.symbolab.com/

https://courses.lumenlearning.com/boundless-algebra/chapter/trigonometry-and-right-triangles/#:~:text=The%20relation%20between%20the%20sides,(sides%20a%20and%20b%20).&text=Side%20b%20is%20the%20side,and%20opposed%20to%20angle%20B%20.

https://www.mathsisfun.com/algebra/trigonometry.html
https://www.mathopenref.com/trigfunctions.html

Airframes discussed:

https://www.nationalmuseum.af.mil/

https://www.nationalmuseum.af.mil/Visit/Museum-Exhibits/

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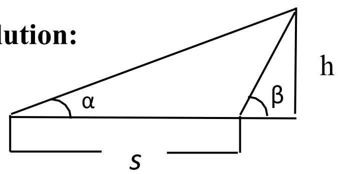
Example Question

Example:

Two tracking stations 12.95 miles apart observe an aircraft approaching their location. What is the aircraft's altitude in ***feet*** if the plane's angle of inclination at the first station is 30° and the angle at the second tracking station is 70°?

**Solution:**




$$h = \frac{s}{\cot a - \cot B}$$

$$h = \frac{12.95}{\cot 30 - \cot 70}$$

$$h = \frac{12.95}{1.7320 - 0.3640}$$

$$h = \frac{12.95}{1.3680}$$

h = **9.4664**

The aircraft's height is 9.4664 miles. To convert this to feet, we know that one mile equals 5,280 feet.

$$\frac{1\ miles}{5{,}280\ feet} = \frac{9.4664}{h}$$

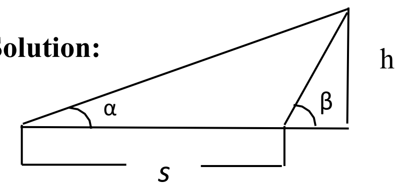
h = 9.4664 x 5,280
h = 49,982.59 feet

The aircraft is 49,983 feet above the ground.

Exercise Questions

**Exercise 1:**
Two tracking stations 20 miles apart observe a B-17 Bomber approaching their location. What is the aircraft's location ***in feet,*** if the plane's angle of inclination at the first station is 15° and the angle at the second tracking station is 60°?

**Solution:**



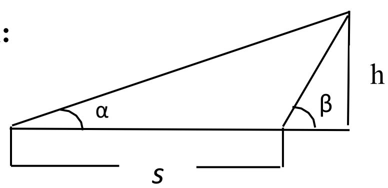

$$h = \frac{s}{\cot a - \cot B}$$

Show all work below

**Exercise 2:**

An SR-71 is flying at 80,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°?

**Solution:**



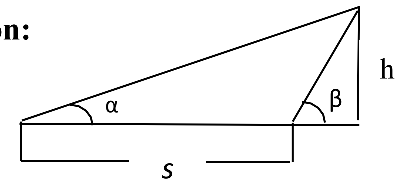

$$\mathbf{h} = \frac{s}{\cot a - \cot B}$$

Show all work below

**Exercise 3:**

A U-2 is flying at 60,968 feet. Two tracking stations are observing it. The tracking stations are 20 miles apart. If the aircraft's angle of inclination at the first station is 30°, what is angle of inclination at the second station?

**Solution:**



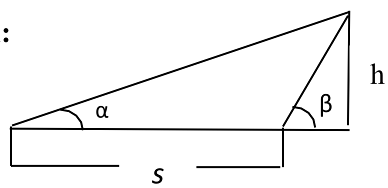

$$\mathbf{h} = \frac{s}{\cot a - \cot B}$$

Show all work below

**<u>Exercise 4:</u>**

A B-2 is returning from a mission flying at 43,462 feet. Two tracking stations are observing it. The tracking stations are 30 miles apart. If the aircraft's angle of inclination at the second station is 85°, what is angle of inclination at the first station?

**Solution:**



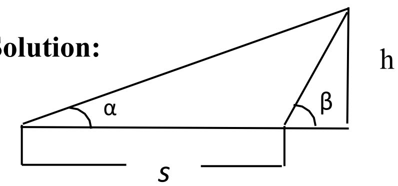

$$\mathbf{h} = \frac{\boldsymbol{s}}{\boldsymbol{cot\ a - cot\ B}}$$

<u>Show all work below</u>

**Exercise 5:**

Two tracking stations 10 miles apart observe a P-26 approaching their location. What is the aircraft's altitude, ***in feet,*** if the plane's angle of inclination at the first station is 20° and the angle at the second tracking station is 85° ?

**Solution:**



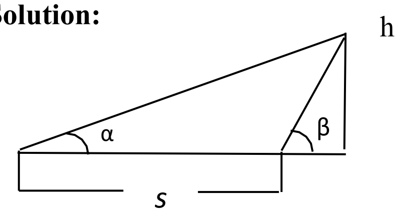

$$\mathbf{h} = \frac{s}{\cot a - \cot B}$$

Show all work below

**Answer Sheet to Exercise Questions**

**Exercise 1:**
Two tracking stations 20 miles apart observe a B-17 Bomber approaching their location. What is the aircraft's location in feet, if the plane's angle of inclination at the first station is 15° and the angle at the second tracking station is 60°?

**Solution:**




$$h = \frac{s}{\cot a - \cot B}$$

$$h = \frac{20}{3.7321 - 0.57735}$$

The aircraft's height is 6.34 miles. To convert this to feet, we know that one mile equals 5,280 feet.

$$\frac{1\ mile}{5{,}280\ feet} = \frac{6.34}{h}$$

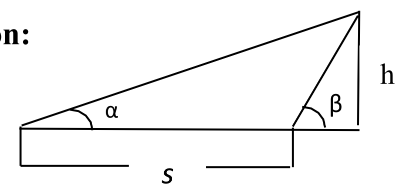
h = 23.27 x 5,280 feet

**h = 33,475 feet**

**<u>Exercise 2:</u>**

An SR-71 is flying at 80,000 feet. Two tracking stations are observing it. How many ***miles*** apart are the tracking stations if the aircraft's angle of inclination at the first station is 10° and the angle at the second tracking station is 80°?

**Solution:**




$h = \frac{s}{\cot a - \cot B}$

$80{,}000 = \frac{s}{\cot 10 - \cot 80}$

$80{,}000 = \frac{s}{5.6713 - 0.1763}$

$80{,}000 = \frac{s}{5.495}$

$5.495 * 80{,}000 \text{ feet} = \frac{s}{5.495} * 5.4950$

s = 439,600 feet

The distance is 439,600 feet

$\frac{1\ mile}{5{,}280\ feet} = \frac{s}{439{,}600}$

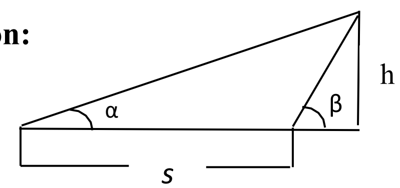
$s = \frac{439{,}600}{5{,}280}$

**s = 83.2576 miles**

**Exercise 3:**

A U-2 is flying at 60,968 feet. Two tracking stations are observing it. The tracking stations are 20 miles apart. If the aircraft's angle of inclination at the first station is 30° and what is angle of inclination at the second station?

**Solution:**




h = $\frac{s}{\cot a - \cot B}$

60,968 = $\frac{20}{\cot 30 - \cot B}$

60,968 (cot 30 – cot B) = 20

60,968*cot(30) – 60,968*cot(B) = 20

20 – 60,968*cot(B) = 20

$\frac{-60{,}968 * \cot(B)}{-60{,}968} = \frac{0}{-60{,}968}$

Cot(B) = 0

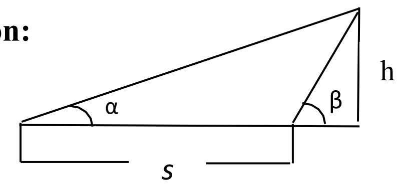
B = arc cotangent (0)

**B = 90°**

**Exercise 4:**

A B-2 is returning from a mission flying at 43,462 feet. Two tracking stations are observing it. The tracking stations are 30 miles apart. If the aircraft's angle of inclination at the second station is 85° and what is angle of inclination at the first station?

**Solution:**




$$h = \frac{s}{cot\ a - cot\ B}$$

$cot\ a - cot\ B$ = s/h

$cot\ a =\ cot\ B$ + s/h

a = arccot ($cot\ B$ + s/h)

$$a = \text{arccot}(\cot(85) + \frac{30 * 5280}{43462})$$

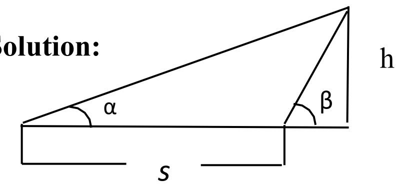
$$\text{Arccot} \left[ \frac{30 * 5280}{43{,}462} + \cot(85) \right] = 14.99999$$

**a = ~15°**

**Exercise 5:**

Two tracking stations 10 miles apart observe a P-26 over them. How high, ***in feet,*** is the aircraft above the ground if the plane's angle of inclination at the first station is 20° and the angle at the second tracking station is 85°?

**Solution:**




$$h = \frac{s}{\cot a - \cot B}$$

$$h = \frac{10}{2.66}$$

$$h = \frac{10}{\cot 20 - \cot 60}$$

h = 3.7594

$$h = \frac{10}{2.7475 - 0.0875}$$

The aircraft's height is 3.7594 miles. To convert this to feet, we know that one mile equals 5,280 feet.

$$\frac{1\ mile}{5{,}280\ feet} = \frac{8.5250}{h}$$

h = 3.7594 x 5,280 feet

**h = 19, 850 feet**

The aircraft is 19,850 feet above the ground.